An Efficient Counting Network

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We present a practical counting network which:

- Is efficient in terms of contention
- Its construction is based on new techniques

We proceed by presenting:

- Background for counting networks
- Our counting network construction
- Remarks and conclusions
Counting Networks

Introduced by Aspnes, Herlihy and Shavit in STOC 91

Distributed data structures used for:

- Shared counters
- Producer/consumer buffers
- Barrier synchronization

Advantages:

- Low contention
- Non-blocking
They look like Sorting Networks

\[ \text{width} = 4 \]

\[ \text{depth} = 3 \]
The **Balancer**

\[
\begin{align*}
  y_0 &= 1 \\
  y_1 &= 0 \\
\end{align*}
\]

\[
\begin{align*}
  y_0 &= 2 \\
  y_1 &= 2 \\
\end{align*}
\]

**Step Property:**

\[0 \leq y_0 - y_1 \leq 1\]
The counting network

Step Property:

\[ 0 \leq y_i - y_j \leq 1 \]

for \( i < j \)
Contention

- Concurrent processors access the same balancer at the same time

Amortized Contention (Dwork et al. STOC 93)

- The number of tokens goes to infinity
Practical counting networks:
  • Bitonic
  • Periodic

Have depth $O(\log^2 t)$

Have amortized contention $O\left(\frac{n\log^2 t}{t}\right)$

$t = $ width
$n = $ concurrency

Problem:
  • To decrease contention we increase depth
Our Counting Network

Has input width $t \leq w$ output width

Balancer

Counting Network
Has depth $O(\lg^2 t)$

Has amortized Contention $O\left(\frac{n\lg^2 t}{w} + \frac{n\lg t}{t}\right)$

$t = \text{input width}$

$w = \text{output width}, \ t \leq w$

$n = \text{concurrency}$

Advantages of increasing output width:

- Depth stays the same
- Contention decreases
Setting \( w = O(t \lg t) \)

we obtain contention \( O\left(\frac{n \lg t}{t}\right) \)

For input width \( t \) it achieves

- Same depth as other practical counting networks of width \( t \)
- Improves contention by a \( \lg t \) factor
The construction

Counting $t, w$

Counting $\frac{t}{2}, \frac{w}{2}$

Merger $\frac{t}{2}, w$
The \textit{Bounded Difference Merger}

Merger $t, w$
Remarks and Conclusions

Output width $p2^k$ (most have width $2^k$)

Extend the construction to arbitrary widths

Use the Bounded Difference Merger for other constructions