1: Taylor’s polynomials (5+5=10 points)

Derive Taylor’s polynomials of degree n for:

\[ f(x) = \sqrt{1 + x} \quad \text{and} \quad f(x) = \cos x \]

Find the approximate value of above functions at \( x = \pi/4 \) by hand calculator upto two decimal points. Show steps.

2: Bisection Method (5+5=10 points)

Find the root of following equations using hand calculator upto two decimal points. Show steps.

\[ x = e^{-x} \]

\[ x^3 - 2x - 2 = 0 \]

3: Errors (10 points)

Evaluate

\[ f(x) = \frac{1}{1 - x^3} - \frac{1}{1 + x^3} \]

over \(-0.00001 \leq x \leq 0.00001\). Increment \( x \) by 0.0000001 at each step in this interval and generate 201 points. Plot the graph of this function. Now, appropriately rewrite the expression to get better graph. Explain the source of error. Submit matlab plot and code.

4: Calculating \( \pi \) using Taylor’s theorem (20 points)

Calculate the value of \( \pi \) upto ten decimal points using the fact that \( \pi = 4 \tan^{-1}(1) \). Use the fact that

\[ \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2} \]

along with the technique in example on page 14 of the text book. First derive the series for \( \tan^{-1}(x) \). Next estimate theoretically and practically how many iterations are required. Use the equation \( \pi/4 = \tan^{-1}(1/2) + \tan^{-1}(1/3) \) to evaluate the value of \( \pi \). How many iterations doe this require? Comment on the difference. Submit matlab code.

5: Root finding (10+5 points)

Use Newton’s method to calculate \( \sqrt{b} \) for a given value \( b \). Obtain the value of \( \sqrt{3} \). Count the number of iterations required to obtain matlab long format accuracy. Also submit matlab code.

Now apply Newton’s method to find the root of \( x^3 - 3x^2 + 3x - 1 = 0 \). Count the number of iterations required to converge upto 10 decimal points accuracy. Comment on any unusual behaviour.