

# Strategic Path Reliability in Information Networks<sup>\*</sup>

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**Abstract:** We consider a model of an information network where nodes can fail and transmission of information is costly. The formation of paths in such networks is modeled as the Nash equilibrium of an  $N$  player routing game. The task of obtaining this equilibrium is shown to be  $NP$ -Hard. We derive analytical results to identify conditions under which the equilibrium path is congruent to well known paths such as the most reliable or cheapest neighbor path. The issue of characterizing off-equilibrium paths in the game is addressed and different path utility metrics proposed. Our first metric measures the degree of individual node suboptimality by evaluating paths in terms of the weakness of the worst-off player. It is shown that there exist information networks not containing paths of weakness less than  $V_r/3$ . Consequently, guaranteeing approximate equilibrium paths of bounded weakness is computationally difficult. We next propose a team game with players having a common payoff function whose equilibrium outcome can be computed in polynomial time. Finally, a *fair* team game with bounded payoff-difference is proposed whose equilibrium is also easily computable.

**Keywords:** Nash Networks, Equilibrium Complexity, Team Games.

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## Abstract

We consider a model of an information network where nodes can fail and transmission of information is costly. The formation of paths in such networks is modeled as the Nash equilibrium of an  $N$  player routing game. The task of obtaining this equilibrium is shown to be  $NP$ -Hard. We derive analytical results to identify conditions under which the equilibrium path is congruent to well known paths such as the most reliable or cheapest neighbor path. The issue of characterizing off-equilibrium paths in the game is addressed and different path utility metrics proposed. Our first metric measures the degree of individual node suboptimality by evaluating paths in terms of the weakness of the worst-off player. It is shown that there exist information networks not containing paths of weakness less than  $\frac{V_c}{3}$ . Consequently, guaranteeing approximate equilibrium paths of bounded weakness is computationally difficult. We next propose a team game with players having a common payoff function whose equilibrium outcome can be computed in polynomial time. Finally, a *fair* team game with bounded payoff-difference is proposed whose equilibrium is also easily computable.

## 1 Introduction

In recent years there has been a growing body of literature that models the behavior of economic agents by means of a network. A network is a graph whose vertices represent the agents and the edges show the links between the different agents. The graph which now completely defines the interaction structure between the agents is used to model different economic phenomena, ranging from buying and selling goods, to international trade and information flows. In this paper we model an information network where the Nash equilibrium defines the optimal information flow path.<sup>2</sup>

The model of information flow networks was introduced by Bala and Goyal (2000a). It models the situations where players derive benefits from having more information. Each

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<sup>2</sup> Network models of economic phenomena like collaborative oligopolies (Goyal and Joshi (2002)) and buying and selling (Kranton and Minehart (2001)) use the notion of *pairwise stability* introduced by Jackson and Wolinsky (1996) as the equilibrium concept.

player can access more information through a process of costly link formation with other players. In a companion paper Bala and Goyal (2000b) allow for links between players to fail with uniform probabilities, thereby converting the problem into one of strategic reliability. Haller and Sarangi (2001) generalize the model by allowing for different link failure probabilities and introduce several variations of the initial model to address potential shortcomings. The focus of these models is on the structure of the network formed and the degree of overlap between the set of equilibrium networks and efficient networks.

In the current paper we introduce a game-theoretic model of strategic reliability to study the formation of equilibrium paths.<sup>3</sup> A source node with valuable information must convey this bit of information to a destination node through a sequence of nodes each of whom can fail. Moreover, information transmission is a costly process and maximizing payoffs must take into account the costs and benefits of alternate paths. This model is of significant importance to a new area of research in computer science called distributed sensor networks. A distributed sensor network is a web of sensors used collectively to perform a wide array of tasks ranging from military applications such as target detection, location and tracking to environmental monitoring and surveillance (Brooks, Griffin and Friedlander (2000)). The key feature of such networks is that the sensor nodes are unattended and untethered (independent). Hence the network must be self-configuring i.e., the nodes must make information routing/connectivity decisions in a decentralized manner. Moreover, communication must be energy efficient since battery power cannot be easily replenished. Current models for communication in these networks use protocols like diffusion routing Intanagonwiwat *et al.* (2001), which uses local gradients to identify paths for sending information. However, these protocols do not optimize network wide reliability in conjunction with minimizing communication costs. Our contribution in this paper is to propose a model that explicitly optimizes over both dimensions. Furthermore, the lack of an existing theoretical framework in which to analyze such information networks often forces researchers to resort to simulations. Theoretical results when they exist are very specific to the model in question. This makes it quite hard to compare models and derive general conclusions.

We believe that game theory can provide the appropriate theoretical framework to analyze distributed information networks. By the very nature of their deployment these networks cannot be controlled at every step by the network designer. This scenario of distributed decision making fits very well with the spirit of game theory. By designing the payoff

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<sup>3</sup>As opposed to equilibria that form *spanning subgraphs* based on edge failure in other game theoretic models of reliability.

function suitably the network designer can achieve different degrees of collaborative tasking among the sensors. Finally, the techniques introduced here can also be adapted to model the trade-offs to intelligent network nodes under other optimization criteria such as throughput or delay and thus will be useful to obtain general conclusions about the operation of such networks.

We develop a model of strategic reliability based on node failure with the objective of finding equilibrium paths. Linial (1994) in his seminal study on the interface between game theory and theoretical computer science posed four questions which he suggested are of common interest to both fields. One of these was to classify basic game-theoretic parameters such as values, optimal strategies, equilibria as being easy, hard or even impossible to compute and wherever possible develop efficient algorithms to this end. We analyze information networks in this paper from the above perspective. Indirectly the paper also provides insight on another question he posed: How does the outcome of a given game depends on the players' computational power? We first show that the task of finding the equilibrium path is itself *NP*-Hard. Then based on the exogenous parameter values (probability of node failure, link costs and the value of information) of the game, we identify conditions under which the equilibrium path coincides with other well known paths like the most reliable path. One important issue that emerges from the analysis is the task of ranking suboptimal paths from individual node perspective. We suggest three ways to characterize suboptimal paths. The first of these ranks paths based on their vulnerability, i.e., the weakness of the weakest node. The second method adapts the idea of a team game suggested by von Stengel and Koller (1997). We model the nodes as a group of players whose common adversary is computational complexity.<sup>4</sup> The group now has a common payoff function and this allows us to develop a simple algorithm. Our final method of ranking paths takes into account ease of computation along with fair use of resources by players in the game.

This paper is organized as follows. Section 2 sets up the basic model. Results are presented in Section 3. In Section 4 we develop path evaluation metrics to rank suboptimal paths. The final section has concluding remarks about future research directions.

## 2 The Model

Let  $S = \{s_1, \dots, s_n\}$  denote the set of players (or sensors), with generic members  $i$  and  $j$ . For ordered pairs  $(i, j) \in S \times S$ , the shorthand notation  $ij$  is used. We assume throughout that

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<sup>4</sup>Linial (1994) suggests treating the faulty nodes as the adversary.

$s_n \geq 3$ . Without loss of generality the source node  $s_r = s_1$  has information of value  $V_r$  which it wishes to send to the destination node  $s_q = s_n$ .  $V_r$  represents an abstract quantification of the value of event information at node  $s_r$ . Information is routed through an optimally chosen set  $S' \subseteq S$  of intermediate nodes by forming links. Link formation occurs by a process of simultaneous reasoning at each node leading to a path from  $s_r$  to  $s_q$ . This link formation is costly with each node incurring a cost for the link it establishes. We denote the cost of link  $ij$  by  $c_{ij} > 0$ .<sup>5</sup> Furthermore, we assume that node  $s_i$  can independently fail<sup>6</sup> with a probability  $(1 - p_i) \in (0, 1)$ . Thus  $G = (S, E, P, C)$  represents an instance of an information network in which information of value  $V_r$  is to be optimally routed from node  $s_r$  to node  $s_q$ , where  $S$  is the set of players interconnected by edge set  $E$ ,  $P(s_i) = p_i$  are the success probabilities and  $C(s_i, s_j) = c_{ij}$ , the cost of links in  $E$ . We denote a path from any node  $s_a$  to  $s_b$  in  $G$  by the node sequence  $(s_a, s_2, \dots, s_b)$ .

In this context, we define the following problem called **Reliable Query Reporting** (RQR): Given that information transmission in the network is costly and not fully reliable, how can we induce the formation of maximally reliable paths in  $G$  from reporting to querying nodes where every node is also maximizing its own payoffs? The solution to this problem lies in designing payoff functions such that the Nash equilibrium of this game corresponds to the optimally reliable path.<sup>7</sup> We now describe the different components of this strategic game.

**Strategies.** Each node's strategy is a vector  $l_i = (l_{i1}, \dots, l_{ii-1}, l_{ii+1}, \dots, l_{in})$  and  $l_{ij} \in \{0, 1\}$  for each  $j \in S \setminus \{i\}$ . The value  $l_{ij} = 1$  means that nodes  $i$  and  $j$  have a link initiated by  $i$  whereas  $l_{ij} = 0$  means that sensor  $i$  does not send information to  $j$ . The set of all pure strategies of player  $i$  is denoted by  $\mathcal{L}_i$ . We focus only on pure strategies in this paper. Given that node  $i$  has the option of forming or not forming a link with each of the remaining  $n - 1$  nodes, the number of strategies available to node  $i$  is  $|\mathcal{L}_i| = 2^{n-1}$ . The strategy space of all agents is given by  $\mathcal{L} = \mathcal{L}_1 \times \dots \times \mathcal{L}_n$ . Notice that there is a one-to-one correspondence between the set of all directed networks with  $n$  vertices or nodes and the set of strategies  $\mathcal{L}$ . In order to keep the analysis tractable, in this model we assume that each node can

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<sup>5</sup>For the application we have in mind, this link cost can be an abstraction of packet transmission costs in terms of required transmission power or available on-field sensor battery life, depending on the type of sensor network being modeled. Keeping this in mind players in the game are sometimes referred to as sensors.

<sup>6</sup>We assume that the destination node  $s_q$  never fails.

<sup>7</sup>Note that our techniques can be adapted to achieve other desired network objectives such as delay or throughput as well.

only establish one link. Routing loops are avoided by ensuring that strategies resulting in a node linking to its ancestors yield a payoff of zero and are thus inefficient. Under these assumptions each strategy profile  $l = (l_1, \dots, l_n)$  becomes a **simple directed path** from  $s_r$  to  $s_q$  denoted by  $\mathcal{P}$ . We now proceed to model the payoffs in this game.

A standard noncooperative game assumes that players are *selfish* and are only interested in maximizing their own benefits. This poses a modeling challenge as we wish to design a decentralized information network that can behave in a collaborative manner to achieve a joint goal while taking individual operation costs into account. Since the communal goal in this instance is reliable information transmission, the benefits to a player must be a function of path reliability but costs of communication need to be individual link costs.

**Payoffs.** Consider a strategy profile  $l = (l_i, l_{-i})$  resulting in a path  $\mathcal{P}$  from  $s_r$  to  $s_q$ . Since every agent has an incentive to ensure information is routed to  $s_q$ , the benefit to any agent  $i$  on  $\mathcal{P}$  must be a function of the path reliability from  $i$  onwards. Since the network is unreliable, the benefit to player  $i$  should also be a function of the expected value of information at  $i$ . Hence we can write the payoff at  $i$  as:

$$\Pi_i(l) = \begin{cases} g_i(V_r)f_i(R) - c_{ij} & \text{if } s_i \in \mathcal{P} \\ 0 & \text{otherwise} \end{cases}$$

where  $R$  denotes path reliability.

Fig. 1 illustrates this idea by looking at two adjacent nodes on a path. The expected value of information at node  $j$  is  $p_i p_j V_i$ , i.e., node  $j$  gets the information only when nodes  $i$  and  $j$  survive with probability  $p_i$  and  $p_j$  respectively. The expected benefit to player  $i$  is given by  $p_j V_i$ , i.e., player  $i$ 's benefits depend on the survival probability of player  $j$ . Hence the payoff to player  $i$  is  $\Pi_i = p_j V_i - c_{ij}$ .



Figure 1: Information transfer on a path.

The payoff function that corresponds to this idea of communal reliability and individual

costs can now be written as follows:

$$\Pi_i(l) = V_r \prod_{t=a}^i p_t \prod_{t=i+1}^q p_t - c_{ij}$$

where  $g_i(V_r) = V_r \prod_{t=a}^i p_t$  and  $f_i(R) = \prod_{t=i+1}^q p_t$ . We also use  $\Pi_i^{\mathcal{P}}$  as the payoff to node  $s_i$  in the strategy profile represented by path  $\mathcal{P}$ .

**Definition 1** A strategy  $l_i$  is said to be a **best response** of player  $i$  to  $l_{-i}$  if

$$\Pi_i(l_i, l_{-i}) \geq \Pi_i(l'_i, l_{-i}) \text{ for all } l'_i \in \mathcal{L}_i.$$

Let  $BR_i(l_{-i})$  denote the set of player  $i$ 's best response to  $l_{-i}$ . A strategy profile  $l = (l_1, \dots, l_n)$  is said to be an **optimal RQR path**  $\mathcal{P}$  if  $l_i \in BR_i(l_{-i})$  for each  $i$ , i.e., sensors are playing a Nash equilibrium. Note that although each agent can form only one link, multiple equilibrium paths can exist. For a given node we assume that if multiple optimal paths with identical payoffs exist, the most reliable among them is chosen.

## 3 Results

This section contains results on two aspects of the RQR problem. We first analyze the complexity of computing the optimally reliable paths in a given sensor network. This is followed by some analytical results that establish congruence between the equilibrium RQR path and other well known (global) path metrics.

### 3.1 Complexity Results

Many of the quantities and parameters studied in game theory can at least in principle be computed and approximated. Determining the existence of efficient algorithms for computing equilibria (and finding such algorithms if they exist) is a problem of common interest to game theory and computer science (Linial 1994). There have been many efforts made to characterize the equilibria of different games in terms of their computational complexity. Gilboa and Zemel (1989) show that finding an equilibrium of a bimatrix game with maximum payoff sum is NP-Hard. Koller and Megiddo (1992) show that finding max-min strategies for two person zero-sum games is NP-Complete in general, but give the first polynomial time algorithm for such games in extensive form. Koller, Megiddo and von Stengel (1996)

extend the above result to non-zero sum games, using a complementary pivoting algorithm. Finding optimal strategies for two person games such as chess and go have been shown to be NP-Hard (see Garey and Johnson (1979) for an exhaustive list of known NP-Complete problems). We now address the complexity of finding the equilibrium of the  $N$ -person RQR game.

Let  $G = (S, E, P, C)$  represent an instance of an information network in which information of value  $V_r$  is to be routed from node  $s_r$  to  $s_q$ . Only those strategy profiles that define a path from  $s_r$  to  $s_q$  are of interest and must be evaluated to compute the optimally reliable path. To compute this path each player calculates a path through a sequence of descendants whose reliability (given similar decisions by descendant nodes) relative to the immediate successor's link cost, is maximum at that node.

**Theorem 1** *Given an arbitrary network  $G = (S, E, P, C)$  with information  $V_r$ , computing the optimal RQR path is NP-Hard.*

**Proof:** Given a solution to the RQR problem, for each node on the path verifying optimality of the successor requires exhaustively checking all possible paths to  $s_q$ . Thus RQR does not belong to the class  $NP$ .

We show that the problem is  $NP$ -Hard by considering a reduction from the Hamiltonian Path problem (see Garey and Johnson (1979) for Hamiltonian Path reduction). Let  $G' = (V', E')$  be any graph in which a Hamiltonian Path is to be found, where  $|V'| = n$ . We convert  $G'$  into another graph  $G = (S, E, P, C)$  on which an instance of RQR with value<sup>8</sup>  $V_r = 1$ , must be computed as shown in Fig. 2.

Introduce  $n + 1$  new vertices to form  $S = V' \cup T \cup s_q$ , where  $|T| = n$  and  $s_q$  is the other new vertex. The new edge set  $E$  consists of the original edge set  $E'$  along with  $n^2$  new edges from  $E_2 = T \times V'$  and  $n$  new edges from  $E_3 = T \times s_q$ . Edges in  $E'$ ,  $E_2$  and  $E_3$  are assigned costs  $c_1$ ,  $c_2$  and  $c_3$  respectively. All vertices  $u \in V'$  and  $w \in T$  are assigned success probabilities  $p_1$  and  $p_2$  respectively. The relationships between the probabilities and costs are as follows:

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<sup>8</sup>We set  $V_r = 1$  for notational simplicity since results for any  $V_r$  can be obtained by scaling edge costs appropriately.

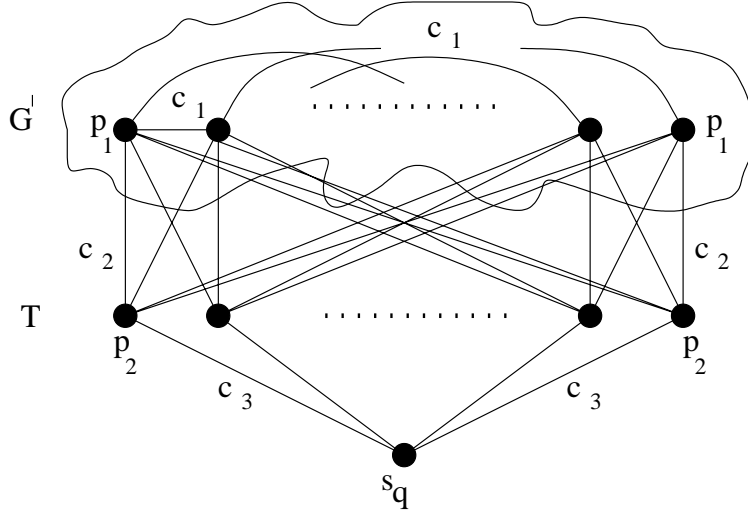


Figure 2: Reduction from Hamiltonian path.

$$p_1 p_2 > \left(\frac{3}{4}\right)^{\frac{1}{n-1}} \quad (1)$$

$$c_1 = \frac{(p_1 p_2)^n}{3} \quad (2)$$

$$c_2 = \frac{2(p_1 p_2)^n}{3} \quad (3)$$

$$c_3 = (p_1 p_2)^n \quad (4)$$

Let  $s_r$  and  $s_t$  be any two nodes in  $V'$ . We claim that there exists an optimal RQR path of reliability  $p_1^n p_2$  from  $s_r$  to  $s_q$  in  $G$  if and only if there exists Hamiltonian path from  $s_r$  to  $s_t$  in  $G'$ .

For the first part of the claim, assume there is a Hamiltonian Path  $\mathcal{H} = (s_r, \dots, s_t)$  in  $G'$ . Consider the path  $\mathcal{H}$  followed by the edges  $(s_t, x)$  and  $(x, s_q)$  in  $G'$ , where  $x$  is any node in  $T$ . This path has reliability  $R(\mathcal{H}) = p_1^n p_2$ . The payoff of node  $s_t$  is  $R(\mathcal{H}) - c_2$  obtained by linking to node  $x$ , which is optimal since there does not exist any other unvisited node in  $V'$ . Similarly the payoff of node  $x$  is also optimal since it can only link to  $s_q$ . Now consider the  $k$ -th node in  $\mathcal{H}$ ,  $1 \leq k \leq n - 1$ . The two choices for this node are either to link to some node  $x \in T$  or the node in  $G'$  that lies on the Hamiltonian path  $\mathcal{H}$ . If the first option is chosen, the most reliable alternate path (and hence the maximum possible alternate payoff) is given by  $p_1^k p_2 - c_2$  which is less than  $R(\mathcal{H}) - c_1$  by conditions (1) – (3). Thus, the second

choice is optimal for this node.

For the second part of the claim, we need to show that if no Hamiltonian path exists in  $G'$ , there cannot be an optimal RQR path of reliability  $p_1^n p_2$ . Note that linking to any available node in  $V'$  with cost  $c_2$  is always preferable for any node  $s_i \in T$ . The worst case payoff to  $s_i$  via a link of cost  $c_2$  is  $p_1^n p_2^n - c_2$ , which outweighs the best possible payoff via a link of cost  $c_3$  which is  $p_1 p_2 - c_3$ . So the optimal path *must* visit all nodes in  $V'$ . To maximize payoffs, the optimal path must have the shortest possible length. This will require minimizing visits to  $T$ . The optimal path will thus consist of sequences of long paths in  $V'$  (the longest possible since any node in  $V'$  will always prefer to link to another node in  $V'$ , if feasible), interspersed with visits to  $T$ . Since  $G'$  does not contain a Hamiltonian path there will be at least two visits to nodes in  $T$  and hence the reliability of such a path will be at least  $p_1^n p_2^2$  which is less than  $p_1^n p_2$  as claimed. ■

It can be seen easily that the above reduction is still valid when all nodes in  $V'$  and  $T$  have the same success probability  $p$ . Consequently, the RQR problem remains  $NP$ -Hard for the special case when nodes have equal success probabilities. The case when all edges have the same cost is much simpler, however, as will be shown below.

### 3.2 Analytical Results

Given the complexity of finding the equilibrium RQR path, we next identify conditions under which this path coincides with other commonly used routing paths. In particular, we look at the most reliable path [MRP] which can be computed using well known techniques such as Dijkstra's shortest path. We also look at paths obtained when nodes select next-neighbors using a localized criterion, i.e., the cheapest neighbor.

Let  $G$  be an arbitrary information network with the source node having value  $V_r$ . Then the following results hold.

**Observation 1** *Given  $p_i \in (0, 1]$  and  $c_{ij} = c$  for all  $ij$ , then the most reliable path always coincides with the equilibrium path.*

**Proof:** Consider the most reliable path from the reporting node  $s_r$  to the destination node  $s_q$ . Clearly, the maximum payoff to  $s_r$  is obtained from this path. Given the assumption of uniform costs the payoff to any other sensor  $s_i \in S$  on this path must also be maximum. Otherwise, it would be possible to find a more reliable path from  $s_r$  to  $s_q$  via  $s_i$ . ■

Note that for uniform  $p_i$ , the equilibrium path also coincides with the cheapest path. Before proceeding further, we now introduce some notation. For any node  $s_i$ , let  $c_{\max}^i = \max\{c_{ij}\}$  and  $c_{\min}^i = \min\{c_{ij}\}$ . Also  $c_{\max} = \max_i\{c_{\max}^i\}$  and  $c_{\min} = \min_i\{c_{\min}^i\}$ . We use  $\mathcal{P}_i^l$  to denote a path of length  $l$  from  $s_i$  to  $s_q$  and benefits along this path by  $\mathcal{P}_i^l$ .

**Proposition 1** *Given  $G$  and  $P(s_i) = p \in (0, 1]$ , for all  $i$ , the most reliable path from  $s_r$  to  $s_q$  will also be the optimal path if*

$$c_{\max}^i - c_{\min}^i < p^m(1 - p)V_r$$

for all  $s_i$  on the most reliable path  $\mathcal{P}_r^m$ .

**Proof:** Consider an arbitrary node  $s_i$  at a distance  $i$  from  $s_r$ . Since we have uniform  $p$ , reliability is now inversely proportional to path length. Let  $l$  be the length of the shortest path from  $s_i$  to  $s_q$ , on which  $s_{i+1}$  is the next neighbor of  $s_i$ . For  $s_i$ ,  $\mathcal{P}_i^l$  is optimal if

$$\begin{aligned} V_r p^{i+l} - c_{ii+1} &> V_r p^{i+l+\lambda} - c_{ij} \quad \lambda = 1, 2, \dots \\ \Rightarrow \frac{c_{ij} - c_{ii+1}}{V_r} &< p^{i+l}(1 - p^\lambda) \end{aligned}$$

where  $s_j$  is a neighbor of  $s_i$  through which there is a simple path of length  $l + \lambda$ . Since  $m = i + l$  on  $\mathcal{P}_r^m$ , the reliability term above is minimized for  $\lambda = 1$ , whereas the cost term is maximized at  $c_{\max}^i - c_{\min}^i$ . ■

Note that the above result identifies sufficient constraints on costs for the most reliable path to also be optimal. The result shows that while the MRP can be costlier than other paths, to be optimal it cannot be ‘too’ much more expensive. From the above result, it also follows that when  $c_{\max} - c_{\min} < p^m(1 - p)$  the MRP coincides with the optimal, thereby providing a *global bound* on costs.

We now look at the situation when the probabilities of node survival are non-uniform. Let  $s_i$  and  $s_{i+1}$  be subsequent nodes on the most reliable path. Denote  $R_i$  be the reliability of the most reliable path from  $s_i$  to  $s_q$  and  $R'_i$  be the reliability along any alternative path from  $s_i$ . Let  $\Delta c_i = c_{ii+1} - c_{ij}$  where  $s_j$  is any neighbor not on the optimal path and  $\Delta R_i$  is defined similarly.

**Proposition 2** Given  $G$  with  $P(s_i) = p_i \in (0, 1]$  and  $C = \{c_{ij}\}$ , the most reliable path from  $s_r$  to  $s_q$  will be optimal if

$$\frac{\Delta c_{i+1}}{\Delta c_i} < \frac{\Delta R_{i+1}}{\Delta R_i}$$

for all  $s_i$  and  $s_{i+1}$  on the optimal path.

**Proof:** Let  $\bar{R}_i$  represent the reliability on the portion of the most reliable path  $\bar{\mathcal{P}}$  from  $s_r$  to  $s_i$ . Since  $\bar{\mathcal{P}}$  is optimal,  $s_i$  cannot benefit by deviating if

$$\begin{aligned} V_r \bar{R}_i R_i - c_{ii+1} &> V_r \bar{R}_i R'_i - c_{ij} \\ \Rightarrow V_r \bar{R}_i &> \frac{\Delta c_i}{\Delta R_i} \end{aligned}$$

It follows that  $V_r \bar{R}_{i+1} > \frac{\Delta c_{i+1}}{\Delta R_{i+1}}$ . Since  $\bar{R}_{i+1} = p_{i+1} \bar{R}_i$ , we have  $V_r p_{i+1} \bar{R}_i > \frac{\Delta c_{i+1}}{\Delta R_{i+1}}$ . This can be rewritten as  $1 \geq p_{i+1} > \frac{\Delta c_{i+1}}{\Delta c_i} \frac{\Delta R_i}{\Delta R_{i+1}}$ , which gives us  $\frac{\Delta c_{i+1}}{\Delta c_i} < \frac{\Delta R_{i+1}}{\Delta R_i}$  as desired. ■

The easiest way to interpret this result is by rearranging the terms so that we can write it as  $\frac{\Delta c_{i+1}}{\Delta R_{i+1}} < \frac{\Delta c_i}{\Delta R_i}$ . Then each fraction can be interpreted as the marginal cost of reliability of deviating from the optimal path. Since each subsequent node on the optimal path has lower expected value of information, this results suggests that the marginal cost of deviation in terms of reliability must be higher for each node's ancestor where the expected value of information is also higher.

We define the cheapest neighbor path [CNP] from  $s_r$  to  $s_q$  as the path obtained by each node choosing its successor via its cheapest link. In a sense, this path reflects the optimal route obtained when each node merely cares about minimizing its local communication costs. The following proposition identifies when it will coincide with optimal path.

**Proposition 3** Given  $G$  and  $P(s_i) = p \in (0, 1)$ , for all  $i$ , the optimal path is at least as reliable as the cheapest neighbor path. Furthermore, the CNP will be optimally reliable if

$$\min\{c^k \setminus c_{\min}^k\} - c_{\min}^k > V_r p^l (1 - p^{t-l})$$

where  $l$  is the length of the shortest path from  $s_r$  to  $s_q$  and  $t$  is the length of the CNP.

**Proof:** Consider an arbitrary node  $s_k$  which is  $k$  hops away from  $s_q$  on the CNP. Clearly, for the CNP to be optimal  $s_k$  should not get higher payoff by deviating to an alternative

path. Also, we do not need to consider alternative paths that have lengths greater than  $k$  to  $s_q$  since that would decrease benefits and the CNP already has the lowest cost edges. Let  $m$  be the path length along the CNP from  $s_r$  to  $s_k$ . For alternative paths of length  $i = 1, \dots, k - 1$ , from  $s_k$  to  $s_q$  to be infeasible, we need

$$c_i > c_o + V_r p^{m+i}(1 - p^{k-i})$$

where  $c_o$  is the edge cost along the CNP, and  $c_i$  the edge cost along alternative paths. By definition, for any node on the CNP  $m + i \geq l$ . Also at  $s_k$  we have  $c_o = c_{\min}^k$ , with  $c_i$  being at most  $\min\{c^k \setminus c_{\min}^k\}$ . Thus, when  $\min\{c^k \setminus c_{\min}^k\} - c_{\min}^k > V_r p^l(1 - p^{k-l})$ , the CNP will coincide with the optimal path. ■

The above proposition illustrates that the CNP does not have to be the most reliable in order to be optimal, it only needs to be sufficiently close. For networks in which some paths are overwhelmingly cheap compared to others, routing along CNPs may be reasonable. However, in networks where communication cost are not dissimilar, routing based on local cost (for instance energy utilization) gradients is likely to be less reliable.

## 4 Individual Payoffs versus Global Outcomes

Our modeling of sensor interaction in game-theoretic terms captures the lack of intervention by a central authority, which is the fundamental operational constraint for such information networks. This attractive feature however, poses its own challenges. Equilibrium paths are computed by individually rational players who maximize their own payoffs. But, once we are off the equilibrium path, it becomes difficult to rank the different sub-optimal paths. For example, a certain sub-optimal path may yield high payoffs for player  $i$  with low payoffs for player  $j$ . In another sub-optimal path, the exact opposite situation may prevail, making it difficult to compare these two paths. Note that traditional approaches to finding ‘good’ paths in such networks use a single distinguishing attribute (a scalar) to rank different paths, like minimizing total cost or overall latency. On the other hand, the game-theoretic formulation yields a vector of payoffs. And it will rarely be the case that a sequence of payoff vectors will satisfy a strict inequality, allowing us to order the different suboptimal paths by comparing them with each other and the equilibrium path. While game theorists have proposed numerous techniques for finding approximate Nash equilibria, these methods are not suitable for our purposes due to a number of reasons. Many of these techniques are only

suitable for two-player games. Another problem is that they are not guaranteed to work in all instances. Finally, we are only interested in pure strategy equilibria, while most of these methods rely on the use of mixed strategies.

In this section, we set out to propose criteria for ranking off-equilibrium paths. Game-theory offers a multitude of ways to refine equilibria, but the work on coarsening the set of equilibria is rather limited. Rationalizability (Bernheim (1984), Pearce (1984)) and *curb* sets (Basu and Weibull (1991), Hurkens (1995)) are among the more popular ideas here and not quite appropriate for the task at hand. Both these approaches use rationality to suggest what ‘other’ strategies the player might consider feasible but neither approach provides a way to rank suboptimal outcomes. The specific objective in the type of information networks modeled here is to find a path from the source to the destination. The task of incorporating game-theoretic notions into such route-determination problems requires us to rank the alternate paths precisely. Here we suggest three ways to rank paths: *path weakness*, *path computability* and *path fairness*.<sup>9</sup>

Our objective is to derive uniform evaluation metrics for ranking off-equilibrium paths in terms of their suboptimality. For this purpose we are interested in defining a scalar that measures the degree of suboptimality of the entire path (in a sense a global outcome of the game) while also accounting for individual node behavior. One measure that achieves this compromise between efficient global path characterization and individual node payoffs is aggregate node payoff on the path. However, this is not a suitable measure since it is possible to find paths whose cumulative payoffs are higher than that of the optimal! Given the underlying premise of decentralized decision making, any path evaluation metric must primarily account for the sub-optimality of *individual* node behavior rather than the aggregate response of nodes on the path.

### I. Path Weakness

We formally define our first path utility metric as follows: Consider any node  $s_i$  on the given path  $\mathcal{P}$ . Let  $\hat{\mathcal{P}}_{i_q}$  be the optimal path to  $s_q$  from  $s_i$  in the subgraph  $G \setminus \{s_r, \dots, s_{i-1}\}$ . Thus  $\hat{\mathcal{P}}_{i_q}$  represents the best that node  $s_i$  can do, given the links already established by nodes  $s_r, \dots, s_{i-1}$ . Define  $\Delta_i(\mathcal{P}) = \Pi_i(\hat{\mathcal{P}}_{i_q}) - \Pi_i(\mathcal{P})$  as the payoff deviation for  $s_i$  under strategy profile  $\mathcal{P}$ .  $\bar{\Delta}(\mathcal{P}) = \max_i \Delta_i(\mathcal{P})$  represents the payoff deviation at the node which is ‘worst-off’ in  $\mathcal{P}$ . What can be said about this parameter for nodes participating in optimal and sub-optimal paths?

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<sup>9</sup>Note that one can easily think of alternate and/or application-specific path evaluation metrics. We believe that the metrics suggested here are most relevant to the RQR problem.

**Observation 2**  $\bar{\Delta}(\mathcal{P}') > 0$  for all non-optimal paths  $\mathcal{P}'$ .

However observe that  $\Delta_i(\mathcal{P}')$ —the weakness of individual nodes on off-equilibrium paths can take both positive and negative values. On the other hand,  $\bar{\Delta}(\mathcal{P}) = 0$  if and only if  $\mathcal{P}$  is the Nash equilibrium path of the game. Thus from a global point of view,  $\bar{\Delta}(\mathcal{P})$  identifies the maximum degree to which a node on the path can gain by deviating. This allowing us to rank the ‘vulnerability’ of different paths. This embodies the idea that a path is only as good as its weakest link. We label this path utility measure *path weakness*.

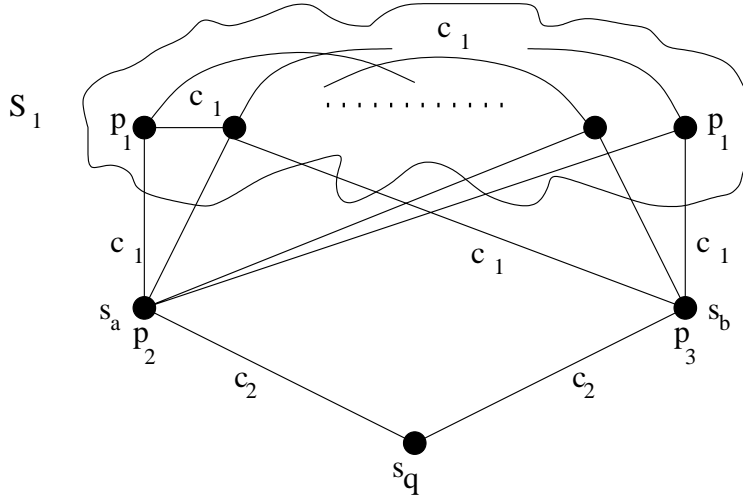


Figure 3: Network for illustrating path weakness.

We now compute bounds for finding paths with low path weakness. We will show that there exist networks not containing paths of bounded weakness. Our proof relies on constructing a specific example of a network whose best suboptimal paths satisfy certain weakness characteristics. This network is constructed below.

Consider an arbitrary sensor network  $G = (S, E)$  as shown in Fig. 3 with the following parameters: The vertex set  $S$  is the union of vertex set  $S_1$  with nodes  $s_a$ ,  $s_b$  and  $s_q$ .  $G' = (S_1, E_1)$  is an arbitrary network, where  $|S_1 = \{s_r = s_1, \dots, s_n\}| = n$ . The edge set  $E$  for  $S$  is the union of disjoint edge sets  $E_1$ ,  $E_2$  and  $E_3$ , where  $E_2 = \{(s_a, s_i)\} \cup \{(s_b, s_i)\}, \forall s_i \in S_1$ , and  $E_3 = (s_a, s_q) \cup (s_b, s_q)$ . There are two types of edge costs in  $C$ —edges in  $E_3$  cost  $c_2$  with all other edges costing  $c_1$ . The node success probabilities are  $P(s_i) = p_1, \forall s_i \in S_1$ ,  $P(s_a) = p_2$  and  $P(s_b) = p_3$ .  $V_r$  is the value of information to be routed from  $s_r$  to  $s_q$ . These parameters are related to each other as follows:

$$p_3 < p_1^{n-2} \quad (5)$$

$$p_3 < p_2 \quad (6)$$

$$p_1 p_2 (1 - p_1 p_3) V_r < c_2 - c_1 \quad (7)$$

$$c_1 < c_2 < p_1^n p_2 p_3 V_r \quad (8)$$

We now look at the strategy choices for nodes in  $G$  on any path from  $s_r$  to  $s_q$ , when receiving  $V_r$ . Condition 8 ensures that all edges in the network are feasible since all payoffs are greater than zero. Also,  $s_q$  is reachable only through  $s_a$  and  $s_b$  and all edges from any node in  $S_1$  have identical costs. Thus if  $s_a$  ( $s_b$ ) is the parent of any node in  $S_1$ , this node will immediately prefer to link to  $s_b$  ( $s_a$ ) to maximize its payoff. Coupled with condition 7, this implies that if node  $s_a$  is visited before  $s_b$  in any path,  $s_a$  prefers to link to any available node in  $S_1$  instead of linking to  $s_q$ , regardless of the number of nodes visited in  $S_1$  prior to  $s_a$ . A similar situation holds true for  $s_b$  if it is visited before  $s_a$ .

Now consider paths  $\mathcal{P}_{ik} = (s_1, \dots, s_k, s_i, s_q)$ , where  $i = a, b$  is the penultimate node for  $k = 1, 2, \dots, n$  and similarly  $\mathcal{P}_k = (s_1, \dots, s_k, s_a, s_{k+1}, s_b, s_q)$ ,  $k = 1, \dots, n-1$ , assuming they exist. The observations above can be used to calculate the path weakness of  $\mathcal{P}_{ak}$  as follows. First,

$$\Delta_a(\mathcal{P}_{ak}) = \begin{cases} V_r p_1^k p_2 (p_1 p_3 - 1) + (c_2 - c_1), & 1 \leq k \leq n-1 \\ 0, & k = n \end{cases} \quad (9)$$

Also, for each node  $s_j$ ,  $1 \leq j \leq k$ ,

$$\Delta_j(\mathcal{P}_{ak}) = \begin{cases} V_r p_1^k p_2 (p_1^{n-k} - 1), & 1 \leq k \leq n-1, \text{ if } \mathcal{P}_{an} \text{ exists} \\ V_r p_1^j p_2 (p_1 p_3 - p_1^{k-j}), & 1 \leq k \leq n-1, \text{ otherwise} \\ 0, & k = n \end{cases} \quad (10)$$

To understand (9)–(10), first note that  $\mathcal{P}_{ak}$  cannot be the equilibrium RQR path whenever  $k < n$ . The optimal choice for  $s_a$  is always to link to any available node in  $S_1$ . Condition

(5) implies that nodes in  $S_1$  would prefer to link to nodes in  $S_1$  and  $s_a$  and avoid visiting  $s_b$  en route to  $s_q$ , if possible. Hence, the optimal payoff for  $s_j$  is via  $\mathcal{P}_{an}$  if it exists, and via the path  $(s_1, \dots, s_j, s_a, s_{j+1}, s_b, s_q)$ , otherwise.

It can be seen that  $\Delta_j(\mathcal{P}_{ak}) \leq 0$ , for all  $j$  and  $k$ . Thus  $\overline{\Delta}(\mathcal{P}_{ak})$ , the path weakness of  $\mathcal{P}_{ak}$ , is given by  $\Delta_a(\mathcal{P}_{ak})$ . Similarly,  $\overline{\Delta}(\mathcal{P}_{bk})$  can be obtained by interchanging  $p_2$  and  $p_3$  in (9).

Now consider paths of type  $\mathcal{P}_k$ ,  $1 \leq k \leq n-1$ .  $\Delta_a(\mathcal{P}_k) = \Delta_{k+1}(\mathcal{P}_k) = \Delta_b(\mathcal{P}_k) = 0$ , since these three nodes are choosing their neighbors optimally. Therefore the path weakness of  $\mathcal{P}_k$  is given by

$$\overline{\Delta}(\mathcal{P}_k) = \Delta_1(\mathcal{P}_k) = \begin{cases} V_r p_1^{k+1} p_2 (p_1^{n-k-1} - p_3), & \text{if } \mathcal{P}_{an} \text{ exists} \\ V_r p_1^2 p_2 p_3 (1 - p_1^{k-2}), & \text{otherwise} \end{cases} \quad (11)$$

Similarly, it can be shown that all paths in which  $s_b$  is visited before  $s_a$  or in which multiple nodes in  $S_1$  are visited in between  $s_a$  and  $s_b$ , are weaker than the above paths.

The following lemma can be used to compute a lower bound on the path weakness of suboptimal paths.

**Lemma 1** *For any  $\epsilon \in (0, \frac{V_r}{3}]$  in the network  $G$ , there exists a path  $\mathcal{Q}$  and probabilities  $p_1, p_2, p_3$ , such that either  $\mathcal{Q}$  is the equilibrium RQR path or  $0 < \frac{V_r}{3} - \overline{\Delta}(\mathcal{Q}) < \epsilon$  and there is no other off-equilibrium path weaker than  $\mathcal{Q}$ .*

**Proof:** Consider all paths  $\mathcal{P} \setminus \mathcal{P}_{an}$  in  $G$ .

$$\min_{\mathcal{P} \setminus \mathcal{P}_{an}} \{\overline{\Delta}(\mathcal{P})\} = \min_k \{\mathcal{P}_{ak}, \mathcal{P}_{bk}, \mathcal{P}_k\}$$

where  $\mathcal{P}_{ak}$ ,  $\mathcal{P}_{bk}$  and  $\mathcal{P}_k$  are as defined before.

Using (6),  $\overline{\Delta}(\mathcal{P}_{bk}) > \overline{\Delta}(\mathcal{P}_{ak})$ , and hence  $\mathcal{P}_{bk}$  is always weaker than  $\mathcal{P}_{ak}$ . Additionally from (9), and (11), it can be seen that

$$\min_{\mathcal{P} \setminus \mathcal{P}_{an}} \{\overline{\Delta}(\mathcal{P})\} = \min \{\mathcal{P}_{a1}, \mathcal{P}_1\} \quad (12)$$

To obtain the result in the lemma, we set  $\mathcal{Q}$  to be the path  $\mathcal{P}_1$  and solve to obtain the corresponding  $p$  values as below.

$$p_1^{n-2} > p_3 > \max \left( \frac{1}{p_1(1 + p_1^{n-2})}, \frac{1 + p_1^{n-1}}{p_1(2 + p_1^{n-2})} \right) \quad (13)$$

The first term in the maximum is obtained using conditions (7) and (8) simultaneously for the network  $G$ . Solving for situations when  $\mathcal{P}_{a1}$  exceeds  $\mathcal{P}_1$  and then using (7) gives the second term. Thus a network  $G$  with the above probability values will have the property that

$$\min_{\mathcal{P} \setminus \mathcal{P}_{an}} \{\overline{\Delta}(\mathcal{P})\} = \overline{\Delta}(\mathcal{P}_1) = \begin{cases} V_r p_1^2 p_2 (p_1^{n-2} - p_3), & \text{if } \mathcal{P}_{an} \text{ exists} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

Identifying the *upper* bound of  $\overline{\Delta}(\mathcal{P}_1)$ , subject to the constraints in (13) yields the desired bounded weakness result. ■

Since  $G'$  is an arbitrary subgraph of  $G$ , the above lemma implies the existence of infinitely many graphs without any suboptimal paths of weakness bounded by  $(\frac{V_r}{3} - \epsilon)$ . Stated another way, path weakness better than  $\frac{V_r}{3}$  is difficult to achieve, as shown in the next result.

**Theorem 2** *There exists no polynomial time algorithm to compute approximately optimal RQR paths of weakness less than  $(\frac{V_r}{3} - \epsilon)$  unless  $P = NP$ .*

**Proof:** Let  $\mathcal{A}$  be an algorithm that outputs a path with weakness at most  $\frac{V_r}{3} - \epsilon$  in polynomial time. For the given  $\epsilon$ , choose  $G$  with  $p_1, p_2, p_3$  as per lemma 1. We can then use  $\mathcal{A}$  as a decision algorithm to solve the Hamiltonian path problem in  $G'$ . If a Hamiltonian path exists in  $G'$ , by lemma 1 we have  $\overline{\Delta}(\mathcal{P}_1) > \frac{V_r}{3} - \epsilon$ . In that case, the only path with weakness less than  $\frac{V_r}{3} - \epsilon$  in  $G$  and therefore output by  $\mathcal{A}$  is  $\mathcal{P}_{an}$ , which of course contains a Hamiltonian path in  $G'$ . Algorithm  $\mathcal{A}$  will return some other path in  $G$  (which can be verified as non Hamiltonian in polynomial time) only if no Hamiltonian path exists in  $G'$ . Thus  $\mathcal{A}$  is a polynomial time decision algorithm for solving the Hamiltonian path problem. This is impossible unless  $P = NP$ . ■

## II. Path Computability

In the previous section we have seen that finding paths of bounded weakness is computationally difficult. While game theory can model the autonomous formation of good routing paths in the network as an equilibrium of rational strategies, the effect of computational complexity on the above process is not easily quantifiable. In this section we take the view that computational complexity is an external adversary inhibiting the ‘easy’ formation of

good routing paths by nodes in the network. What then is the best that cooperating agents can do in the face of this adversary?

The idea of a team of players cooperating against a common adversary to maximize their payoffs can be found in the work of von Stengel and Koller (1997). Similarly, worst case analysis of computer algorithms assumes the existence of an adversary who is free to choose the worst possible input at each instance. One can minimize the adverse influence of computational complexity in finding equilibrium paths by considering a group version of the RQR problem. As before, sensors try to select a routing path that maximizes their payoff. However, to model the worst case outcome, the adversary is allowed to select any of these payoff functions as the group payoff which is to be shared by the entire set of participating nodes. Thus players must now cooperate in the face of the adversary to maximize the common payoff of the group. This notion is similar to the concept of maxmin equilibria in common adversarial games introduced by von Stengel and Koller (1997). Also, aligning the interests of the players such that they have a common optimization objective, confers the added benefit of making this equilibrium easy to compute, as shown below.

We formally define a team version of the RQR problem labeled as the ***Group-RQR*** problem as follows. Consider an information network  $G = (S, E, P, C)$  with information of value  $V_r$ . The payoffs to nodes in the network are defined as follows:

$$\Pi_i(l) = \begin{cases} V_r \prod_{t=r}^q p_t - \max_{(s_i, s_j) \in \mathcal{P}} c_{ij} & \text{if } s_i \in \mathcal{P} \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

All nodes on the path share the payoff of the worst-off node on it. Rather than selecting a neighbor to maximize their individual payoffs as in the original game, nodes in the group-RQR model compromise by maximizing their least possible payoff. Note that this shared payoff effectively defines a group benefit. The Nash equilibrium of the group-RQR game is the path from source to destination containing the node with the highest least cost-reliability trade-off over all paths. In case of multiple equilibria, the path with highest reliability is selected.

Note that the group-RQR problem bears some similarity to the *bottleneck shortest path* problem, which minimizes the cost of the longest edge on the path from the source to the destination node. Indeed, the optimal group RQR path can be interpreted as the bottleneck path to node  $s_q$  with the highest path reliability. Let  $\overline{\mathcal{P}}_c$  represent the most reliable path from  $s_r$  to  $s_q$  that does not traverse any link exceeding cost  $c$ . Then  $\overline{\mathcal{P}}$ , the equilibrium path

of the group-RQR game is given by

$$\bar{\mathcal{P}} = \arg \max_{c_i \in C} \left\{ V_r R(\bar{\mathcal{P}}_{c_i}) - c_i \right\} \quad (16)$$

for each distinct edge cost  $c_i$  in  $C$ . An intuitive technique for computing the optimal group-RQR path is to repeatedly determine the most reliable path in the graph that is obtained by successively removing edges of decreasing distinct cost. In the worst case  $m$  most reliable path calculations are made, where  $m$  is the number of distinct edge costs in the network. Details of a simple algorithm for computing the optimal path can be found in the the appendix.

### III. Path Fairness

We now introduce the notion of path fairness. The optimal group-RQR path maximizes the payoff of the node with the lowest payoff. However, in terms of individual payoffs as defined in the original RQR model, the payoff of the best-off node on the group-RQR path can be arbitrarily larger than the team payoff. For information networks of the type studied in this paper, one can assume the existence of an external network controller who is interested not only in the autonomous formation of reliable routing paths but also in the continued operation of the network. Since the network nodes are assumed to have a limited amount of (non-replenishable) energy for communication, continuous operation of the network would seem to depend on the various (active) network components spending approximately the same amount of energy for communication.

One can thus construe a measure of *path fairness* in which the disparity in individual node payoffs is minimized at the equilibrium outcome. This can be viewed as contributing to the longevity of the sensor networks overall operation, since all nodes on the path are consuming approximately the same energy for their link costs. Since acheiving fairness at the cost of reliability is against the overall path routing objective, this equilibrium should also satisfy the team notion of the RQR game (with the added benefit of easy computability). Thus individual payoffs should be as close to the team payoff as possible.

Formally, the payoffs to nodes in the network in the fair-group-RQR game are defined as follows:

$$\Pi_i(l) = \begin{cases} \frac{V_r \prod_{t=r}^q p_t - \max_{c_{ij} \in \mathcal{P}} c_{ij}}{\max_{c_{ij} \in \mathcal{P}} c_{ij} - \min_{c_{kl} \in \mathcal{P}} c_{kl}} & \text{if } s_i \in \mathcal{P} \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

Thus the equilibrium path is the one which achieves the highest group payoff with the least payoff difference. This path can be found by repeatedly determining the optimal group-RQR path that does not pass through edges greater than a given maximum and smaller than a given minimum cost. Let  $\tilde{\mathcal{P}}_{c_1}^{c_2}$  represent the most reliable path from  $s_r$  to  $s_q$  that does not traverse any link exceeding cost  $c_1$  or lower than cost  $c_2$ . Then

$$\overline{\mathcal{FP}} = \arg \max_{c_i \leq c_j \in C} \left\{ \frac{V_r R(\tilde{\mathcal{P}}_{c_i}^{c_j}) - c_i}{c_i - c_j} \right\} \quad (18)$$

is the equilibrium path of the fair-group-RQR game.

## 5 Conclusion

We introduce game-theoretic techniques to model intelligent behavior in an information network. The formation of reliable communication paths from a source to a destination node are analyzed. Certain extensions of the model suggest themselves immediately. The simultaneous presence of multiple source-destination pairs of sensors is the most obvious one. Dynamic models of network formation usually consider myopic players. In a recent paper Watts (2002) looks at the formation of circle networks assuming non-myopic players. We believe that studying the dynamic evolution of paths in an information network with non-myopic nodes failing over time would also be an useful extension. Other interesting versions of the problem could incorporate uncertainty and localized information. For instance, each player could perhaps be aware only of the failure probabilities and costs of link formation in a neighborhood set of nodes. Decisions made under these constraints could lead to dramatically different results from the full information model analyzed here. Uncertainty in the model could be of the form where a player is only aware of the probability distribution from which link formation costs are drawn instead of knowing these costs precisely. We believe these extensions would be of great practical interest.

The other direction for future research would be to focus on the complexity aspect of the problem. An important task in this context is to develop axiomatic path characterisations for evaluating suboptimal paths. As already argued, a path metric that maximizes the payoff function of one node could very well minimize the payoff function of another node. In the current paper we suggest several alternative techniques to rank sub-optimal paths. The first of these rank paths by determining the weakest link in the communication chain. The second method provides a means to obtain a computationally easy path. The final method ranks

paths on the basis of equitable resource use modeled through the costs of forming links. The three methods suggested here are by no means the only possible ones. We believe that this area needs to be explored further in order to model similar problems in computer science using game-theoretic concepts.

Finally, the techniques introduced here can also be used to model certain types of behavior on the *Internet*. One example could be the reliable communication of information between different web servers. Alternative payoff functions based on different link-establishment criteria also need to be explored in this context. Congestion control, pricing of links and multicast issues are all possible application areas of the game-theoretic formulation introduced here.

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# Appendix

A simple algorithm for computing the optimal group-RQR path is shown below.

## Group-RQR Path Algorithm

1. Let  $G = (S, E, P, C)$  be a sensor network of value  $V_r$  in which the group-RQR path from  $s_r$  to  $s_q$  is to be found.  
Let  $C = \{c_1, c_2, \dots, c_m\}$  be the set of *distinct* edge costs in  $G$  sorted in decreasing order.
2. For  $\mathbf{i} = \mathbf{1}$  to  $\mathbf{m}$ 
  - (a) Compute  $\overline{\mathcal{P}}_{c_i}$  in  $G$  using standard shortest path techniques such as Dijkstra's algorithm.
  - (b)  $\mathcal{P}^{\text{GRQR}} = \max\{\mathcal{P}^{\text{GRQR}}, V_r R(\overline{\mathcal{P}}_{c_i}) - c_i\}$ .
  - (c)  $G = G \setminus c_i$