CSC 4890 – Introduction to the Theory of Computation

Homework 6

Due: Thursday December 6, beginning of class.

Problem 1. (Enumerators) Prove that a language L is decidable *if and only if* there is an enumerator that prints the strings of L in proper (canonical) order.

Problem 2. (Undecidable) Formulate the following problem as a language and then prove that it is *undecidable*:

> Given a Turing machine M a string w and a symbol x, determine whether M writes symbol x on the tape during the computation on input string w.

Hint: use a similar reduction as in the state-entry problem which we described in class.

Problem 3. (Undecidable) Formulate the following problem as a language and then prove that it is *undecidable*:

> Given a Turing machine M determine whether $L(M) = \{have, a, nice, day\}$.

Hint: use a similar reduction as in the Empty, Regular, and Size-2 language problems which we described in class.

Problem 4. (P) Prove that the class P is closed under intersection. That is, show that if $L_1 \in P$ and $L_2 \in P$ then $L_1 \cap L_2 \in P$.

Problem 5. (NP) Consider the vertex coloring problem. In this problem we are given some graph $G = (V, E)$ (where V is the set of nodes and E are the edges) and an integer k, and we are asked to determine if there exists an assignment of a *color* to each vertex, namely, an integer in the range $1...k$, so that no two adjacent vertices have the same color. Show that this problem is in NP.