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FUZZY LOGIC IN DIGITAL MAMMOGRAPHY: ANALYSIS OF LOBULATION

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Abstract: This paper illustrates how the fuzzy logic approach can be used to formalize the American College of Radiology (ACR) breast imaging reporting lexicon. In current practice radiologists make a relatively subjective determination for many terms from the lexicon related to breast cancer diagnosis. Lobulation and microlobulation of nodules are important features in breast cancer diagnosis based on mammographic analysis by using the ACR lexicon. We offer an approach for formalizing the distinction of these features and also formalize the description of the intermediate cases between lobulated and microlobulated masses. In this paper it is shown that fuzzy logic can be an effective tool in dealing with this kind of problems. The proposed formalization creates a base for the next two steps: (i) the automatic extraction of the related primitives from the image, and (ii) the detection of lobulated and microlobulated masses based on these primitives.

Topic Category: *Decision analysis*

Keywords: *fuzzy logic, feature formalization, breast cancer, image recognition.*

1. Introduction

Current methods in digital mammography [1], [8] are mostly based on neural networks without

incorporating fuzzy logic. Nevertheless, it should be mentioned that these methods use degrees of irregularity and circularity which are very close to some key concepts in fuzzy logic. These degrees are used as inputs to neural networks [9]. In this paper we apply a fuzzy logic approach for classifying a mass found in a mammogram as lobulated or microlobulated. The lobulated and microlobulated features of a mass are important in breast cancer diagnosis [7]. The proposed formalization creates a base for the next two steps: (i) the automatic extraction of the related primitives from the image, and (ii) the detection of lobulated and microlobulated masses based on these primitives. We study these next steps in a separate paper [6].

The proposed analysis is based on the definitions of the previous two terms, as given by the American College of Radiology (ACR) lexicon. According to this lexicon a mass has "lobular" shape if "it has contours with undulations." Note that the lexicon defines the notion "lobular" without any indication of the size or number of undulations. The above situation, for a formal computer algorithmic analysis, means that if a mass has any one of "small/medium/large undulation", then the algorithm should classify it as lobular. However, this is not necessarily what occurs in a real life situation. Often, a radiologist takes into account the size, the number of undulations, and how deep they are.

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However, the ACR lexicon does not mention these attributes in the formal definition of lobulation. Therefore, it is likely that different radiologists may have different perceptions about the size and number of undulations sufficient to classify the shape of a mass as lobular or microlobular.

Next, we analyze the term "*microlobulated margins*." This term means (according to the ACR lexicon) that "*the margins undulate with short cycles producing small undulations*." Again, different radiologists may have different perceptions of what "*short cycles*" and "*small undulation*" mean. However, currently the ACR lexicon does not provide a unified framework for defining these terms in a consistent and objective manner. The following two hypothetical examples highlight the need for a unified framework for defining terms related to the shape of masses in mammograms.

Example 1:

Suppose that a radiologist has found one "big" and two "small" undulations in a given mass. Does it mean that this mass is lobular or microlobular or do features of both coexist? Also suppose that a second radiologist has decided that there are two "big" and one "small" undulations for the same mass. Now we have the same question: "*Is this mass lobular or microlobular or do features of both coexist?*".

Example 2:

Suppose that in some study five out of ten radiologists concluded that a particular mass is lobular, but the other five came to the opposite conclusion. How should we train a computer system to detect a lobular mass by utilizing this contradictory experience? Should we exclude the mentioned cases from the training set? We certainly could. However, similar cases may appear again in a real diagnosis problem. If we exclude these cases, any trained detection system will diagnose them arbitrarily, although, most properly it should not identify lobular features.

The last example illustrates a typical source of intra- and extra-observer variability in mammography and its consequences. How can we reduce these problems? We propose a lobular mass detection algorithm which addresses this methodological and practical problem. This algorithm can also become the basis for analyzing and formalizing the rest of the ACR lexicon terms. The proposed algorithm is designed in a way which copies the way human experts make decisions.

Therefore, this paper will concentrate only on the development of an algorithm for formalizing lobularity and microlobularity in the masses found in a mammogram. This paper has the following structure. First it discusses the development the features, which characterize lobularity and microlobularity and formalizing of these features with fuzzy logic approach (section 2). In section 3 it develops the notions of degrees of lobularity and microlobularity based on formalized features. In section 4 it presents a brief description of an experimental testing of formalizing the criteria of lobulation/microlobulation. We finish the paper with some concluding remarks.

2. Formalization with Fuzzy Logic

In this section we slightly change the previous two definitions. We define a mass to be lobular if it has a contour with **some big and deep** undulations. The margins of a mass are microlobulated if they have **several short** cycles producing **several small and shallow** undulations. At a first glance it may appear that we did not move to more accurate definitions. However, these reformulations are of critical importance. They allow us to apply fuzzy logic and express the original two principal ACR definitions as functions of secondary and easily fuzzifiable terms.

The above considerations involve two important fuzzy terms, namely the terms "*some*" and "*several*." These terms have a rather clear meaning when they are used in a context with other terms of natural language [2], [3]. In this way we can define a fuzzy set with the fuzzy terms {*few*, *some*, *several*, *many*} for the **number of undulations**. Note that the number of undulations can be equal to 0,1,2,3,...etc.

For instance, for the fuzzy term "*few*" the number of undulations can be set equal to 0. That is, the corresponding family of the four fuzzy membership functions are: $\mu_{\text{few}}(x)$, $\mu_{\text{some}}(x)$, $\mu_{\text{several}}(x)$, and $\mu_{\text{many}}(x)$ (see also figure 1). For instance, some possible sampled values of these membership functions are: $\mu_{\text{few}}(2)=1/3$, $\mu_{\text{some}}(2)=2/3$, $\mu_{\text{some}}(3)=1$, $\mu_{\text{many}}(2)=0$, etc. Some interviewed radiologists felt comfortable with this formalization. Although one may argue with the numerical values of the above membership functions, the main issue is that it is possible in this way one to quantify fuzzy concepts which are critical in the classification of masses as lobular or microlobular.

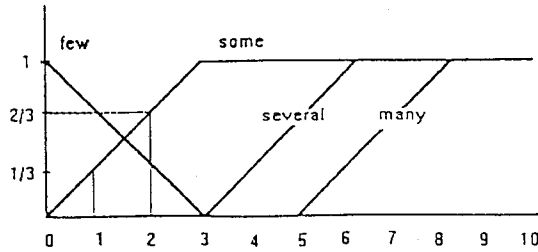


Figure 1. Membership functions related to the number of undulations.

Next we define the meaning of the terms of the fuzzy set $\{small, big\}$. This set is crucial in defining the **size of undulations**. First we need an adequate scale to measure the length of a given undulation. We consider the length of an undulation in **relative** terms since different masses may have different sizes. For instance, an undulation 3 mm in length could be considered as microlobular in a large mass while in a small mass with the same undulation could be considered as lobular.

Therefore we first need to compute L ; the maximum length of a mass. This approach allows to estimate the undulation length as a fraction of L . In figure 2(a) we present a mass with undulations. Specifically, the curve between points A and B is an undulation. Now we can formalize the fuzzy terms "small" and "big" undulations on the scale of the relative undulation length (see also figure 2(b)). According to the membership functions in figure 2(b), a relative length of more than $L/4$ can be defined as a big undulation, while an undulation of relative length of less than $L/12$ could be considered as a small undulation. Undulations of intermediate length can be assigned intermediate membership values.

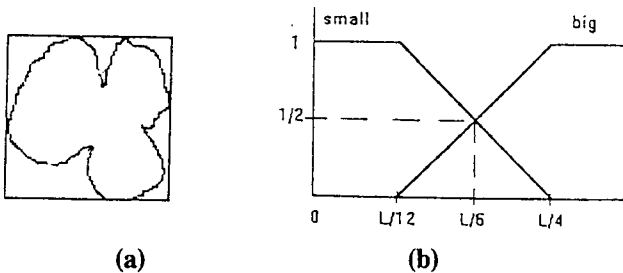


Figure 2. (a) Mass with undulations. (b) Membership functions related to the length of undulations.

We can also define the fuzzy membership functions regarding the deepness ("shallow" or "deep") of undulations. Different masses may have different deepness of lobularity. Thus, we introduce a relative measure of the **deepness of lobularity**, which is defined as a fraction of the maximum length (denoted as L) of the mass. This study is similar to the ones described in the cases of the previous fuzzy sets (and are explained in more detail in the next section).

Therefore, the concept of a lobular mass can be formulated now as follows: **A mass is lobular if it has at least 3 undulations with length and deepness of no less than $L/4$.** Now we can also formulate the concept of microlobulated mass margins. **The mass margins are microlobulated if there are at least 6 undulations with length and deepness of no more than $L/12$.** These definitions allow us to quantify these concepts objectively and consistently. We present the fuzzy logic structures for the lobular and microlobular concepts in figures 3 and 4, respectively.

--> undulation 1 length	= big	$\mu_{big}(undulation1) = 1.00$
deepness	= deep	$\mu_{deep}(undulation1) = 1.00$
MASS --> undulation 2 length	= big	$\mu_{big}(undulation2) = 1.00$
deepness	= deep	$\mu_{deep}(undulation2) = 1.00$
--> undulation 3 length	= big	$\mu_{big}(undulation3) = 1.00$
deepness	= deep	$\mu_{deep}(undulation3) = 1.00$

Figure 3. Fuzzy logic structures for a lobular mass.

--> undulation 1 length	= small	$\mu_{small}(undulation1) = 1.00$
deepness	= shallow	$\mu_{shallow}(undulation1) = 1.00$
--> undulation 2 length	= small	$\mu_{small}(undulation2) = 1.00$
deepness	= shallow	$\mu_{shallow}(undulation2) = 1.00$
MASS --> undulation 3 length	= small	$\mu_{small}(undulation3) = 1.00$
deepness	= shallow	$\mu_{shallow}(undulation3) = 1.00$
--> undulation 4 length	= small	$\mu_{small}(undulation4) = 1.00$
deepness	= shallow	$\mu_{shallow}(undulation4) = 1.00$
--> undulation 5 length	= small	$\mu_{small}(undulation5) = 1.00$
deepness	= shallow	$\mu_{shallow}(undulation5) = 1.00$
--> undulation 6 length	= small	$\mu_{small}(undulation6) = 1.00$
deepness	= shallow	$\mu_{shallow}(undulation6) = 1.00$

Figure 4. Fuzzy logic structures for a microlobulated mass.

Figure 3 shows a mass with three undulations. Each undulation is presented with its length and deepness. All these undulations are big and deep. Hence, all membership functions are equal to 1.00 and this mass is lobular according to our

formalization. Similarly, figure 4 shows a microlobulated mass with 6 undulations and all of them are small and shallow.

The previous definitions allow some masses to be classified as **lobular and microlobulated** without any contradiction if they have at least 9 undulations (3 lobular and 6 microlobular). That is, one just needs to join the structures given in figures 3 and 4. Next, we propose a formal way for dealing with cases which are of intermediate nature. Some examples of such cases are depicted in figure 5.

We take the three biggest and deepest undulations and compute the minimum of their membership function values for the terms "big" and "deep". We define this value as the degree of lobularity (or *DL*). For instance, for the mass described in figure 5 the minimum for the first three undulations is 0.70, that is, for this case $DL = 0.70$. Similarly, it can be easily verified that the degree of microlobularity (or *DM*) computed with the remaining 6 undulations is 0.60. These estimates can be used as some of the inputs to a breast cancer computer-aided diagnostic (CAD) system.

---	undulation 1 length	= big	$\mu_{big}(\text{undulation1}) = 0.80$
	deepness	= deep	$\mu_{deep}(\text{undulation1}) = 0.70$
---	undulation 2 length	= big	$\mu_{big}(\text{undulation2}) = 0.73$
	deepness	= deep	$\mu_{deep}(\text{undulation2}) = 0.71$
---	undulation 3 length	= big	$\mu_{big}(\text{undulation3}) = 0.90$
	deepness	= deep	$\mu_{deep}(\text{undulation3}) = 0.80$
MASS ---	undulation 4 length	= small	$\mu_{small}(\text{undulation4}) = 0.90$
	deepness	= shallow	$\mu_{shallow}(\text{undulation4}) = 0.80$
---	undulation 5 length	= small	$\mu_{small}(\text{undulation5}) = 0.90$
	deepness	= shallow	$\mu_{shallow}(\text{undulation5}) = 0.70$
---	undulation 6 length	= small	$\mu_{small}(\text{undulation6}) = 0.60$
	deepness	= shallow	$\mu_{shallow}(\text{undulation6}) = 0.70$
---	undulation 7 length	= small	$\mu_{small}(\text{undulation7}) = 0.67$
	deepness	= shallow	$\mu_{shallow}(\text{undulation7}) = 0.97$
---	undulation 8 length	= small	$\mu_{small}(\text{undulation8}) = 0.80$
	deepness	= shallow	$\mu_{shallow}(\text{undulation8}) = 1.00$
---	undulation 9 length	= small	$\mu_{small}(\text{undulation9}) = 0.84$
	deepness	= shallow	$\mu_{shallow}(\text{undulation9}) = 0.79$

Figure 5. Structural descriptions for a fuzzy lobular and microlobulated mass.

If the number of undulations is less than 3, we combine the membership functions for the length and deepness with a membership function for the number of undulations (as defined in figure 1). This combination means that now we compute the minimum of these three values in accordance to fuzzy logic practice. We analyze arguments for the minimum in the next section. Now let us consider, for instance, the mass with two undulations presented in figure 6.

---	undulation 1 length	= big	$\mu_{big}(\text{undulation1}) = 0.80$
MASS	deepness	= deep	$\mu_{deep}(\text{undulation1}) = 0.70$
---	undulation 2 length	= big	$\mu_{big}(\text{undulation2}) = 0.60$
	deepness	= deep	$\mu_{deep}(\text{undulation2}) = 0.60$

Figure 6. Structures for a mass with less than three undulations.

Figure 6 allows to compute 0.60 as the corresponding degree of lobularity (*DL*), while figure 1 shows that $\mu_{some}(2) = 0.66$ for a case with 2 undulations. Thus, their minimum of 0.60 characterizes the lobularity of this mass. In the next section we present these ideas formally.

3. Degrees of Lobularity and Microlobularity

Radiologists use an informal approach in determining the lobularity and microlobularity of a mass. To maintain consistency in these evaluations and increase objectivity, we need to formalize these concepts. Let us first consider the two masses depicted in figure 7. Intuitively, the first mass has deep undulations, while the second mass has shallow undulations. Different measures can be created to formalize this distinction. Figure 7(a) shows two distances d_1 and d_2 , defined between the points *A* and *C* and between the points *B* and *E*, respectively, for undulation 1. If each of them is no less than $L/4$, then the undulation is deep (see also figure 7(b)). If these distances are no more than $L/12$, then undulation 1 is shallow (see also figure 7(a)). This means that formally the *deepness D* of the undulation is the pair d_1 , and d_2 . The question of how to compute these values was considered in [6].

We need to compute the values of $\mu_{deep}(d_1)$ and $\mu_{deep}(d_2)$ by using the corresponding membership function in figure 2 in order to transform the previous two measures into a single degree of lobularity for a given undulation. Recall that we use the same membership functions for the length and deepness of undulations. This is done by substituting the terms, "big" for "deep" and "small" for "shallow". Next we compute $\min\{\mu_{deep}(d_1), \mu_{deep}(d_2)\}$, which we consider as the **degree of deepness of the undulation**. That is,

$$\mu_{deep}(\text{undul.}) = \min\{\mu_{deep}(d_1), \mu_{deep}(d_2)\}.$$

Similarly, we define the **degree of shallowness of an undulation** as:

$$\mu_{shallow}(\text{undul.}) = \min\{\mu_{shallow}(d_1), \mu_{shallow}(d_2)\}.$$

Observe that the length of undulation1 (U_1) is measured as the length of the mass margin between

points *A* and *B* (see also figure 7(a)).

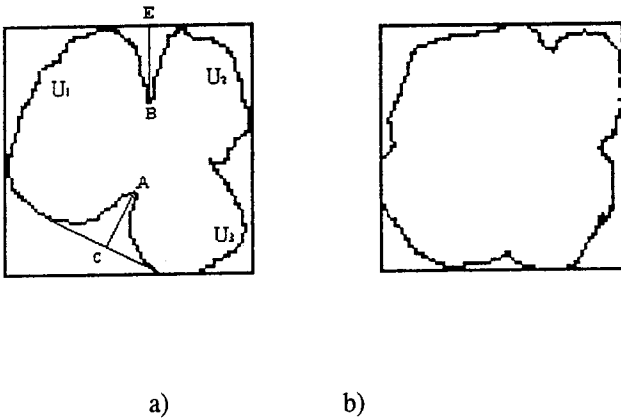


Figure 7. Masses with deep and shallow undulations.

Now we can define the **Degree of Lobularity (DL)** of a mass as follows:

$$DL(\text{mass}) = \min\{\mu_{\text{several}}(k), \min_{k \geq 1} 1\{\mu_{\text{big}}(U_i), \mu_{\text{deep}}(U_i)\}\}, \quad (1)$$

where U_1, U_2, \dots, U_k are undulations such that:

$$\min_{k \geq 1} 1\{\mu_{\text{big}}(U_i), \mu_{\text{deep}}(U_i)\} \geq 0.50.$$

Similarly, we define the **Degree of Microlobularity (DM)** of a mass with k undulations:

$$DM(\text{mass}) = \min\{\mu_{\text{several}}(k), \min_{k \geq 1} 1\{\mu_{\text{small}}(U_i), \mu_{\text{shallow}}(U_i)\}\}, \quad (2)$$

where U_1, U_2, \dots, U_k are undulations such that:

$$\min_{k \geq 1} 1\{\mu_{\text{small}}(U_i), \mu_{\text{shallow}}(U_i)\} \geq 0.5.$$

For the extreme case $k=0$ we have $\mu_{\text{some}}(k)=0$ and $\mu_{\text{several}}(k)=0$ (see figure 1). Therefore, both degrees of lobularity and microlobularity are equal to 0, i.e., the outcome corresponds to what is expected with common sense.

Next, we investigate arguments to justify formulas (1) and (2). There are some general theoretical and experimental arguments for the general case (e.g., [5], [4], [2]), but now we also have some other arguments derived from this mammographic problem. A consistent computer-based breast cancer diagnostic system should refuse to diagnose a mammogram with a significant number of doubtful features. We express how doubtful a given feature is with some degree between 0 and 1. The most doubtful cases have degrees about 0.50. The values of *DL* and *DM* are examples of such degrees.

For these doubtful uncertain features, a CAD system can suggest a diagnosis, but only with some

degree of reliability of this diagnosis. This reliability can be very low. Also, this **degree of reliability** depends on the particular values of *DL* and *DM* degrees. Therefore, the formulas used to define *DL* and *DM* become even more critical. We explain this situation with a modified example from figure 3. Now the first five membership functions for undulations are equal to 1.00 and only the sixth function is not equal to 1.00 (i.e., $\mu_{\text{deep}}(\text{undulation3})=0.60$). Then formula (1) gives us a "pessimistic" estimate, i.e., low degree of lobularity, $DL=0.6$. Substituting in (1) the minimum operation for the maximum will give us an "optimistic," i.e., high degree of lobularity, $DL=1.00$ for this case. In the last "optimistic" estimate we **ignore and lose the warning information** (i.e., the fact that $\mu_{\text{deep}}(\text{undulation1})=0.60$). The value 0.60 shows us that we should be more careful and study this case in more detail not to miss a suspicious case. However, no warning information is lost if we use the "pessimistic" min operation in (1) and (2). Therefore, for critical cancer diagnosis, we see that the "pessimistic" strategy is more consistent. We also consider the diagnosis with a low degree of reliability as a very preliminary diagnosis. This diagnosis shows that we need to **switch the set of features to a higher level of detail** to fully evaluate the complexity of a given case. Many experiments [1], [9] show that relatively simple cases can be diagnosed within a small feature space. For more complicated cases we need a pathologically confirmed training sample with more features and a specifically designed diagnostic method. The whole CAD system which we design will have switching capabilities based on the described approach.

4. An Experiment

With the given formalization we arranged an experiment to test the correspondence of the formalized criteria with radiologists' perception of studied lobulated and microlobulated features. Using our formal criteria, we generated four categories of lobulated and microlobulated mass images: (1) lobulated but not microlobulated; (2) microlobulated but not lobulated; (3) lobulated and microlobulated; (4) not lobulated and not microlobulated. Some of these images were artificially constructed and some were extracted from real mammograms. We accomplished the experiment in two modifications: (i) mass size close to 1×1 cm and (ii) enlarged masses up to the window 6×6 cm. The general conclusion from these experiments is that the formalization is

consistent with experienced radiologists' practice. Detailed results of the experiments are presented in [6].

5. Conclusions

Radiologists often make relatively subjective determination for many features related to breast cancer diagnosis in current practice. We formalized some important features from the ACR breast imaging lexicon, i.e., lobulation and microlobulation of nodules. This formalization creates a base for the next two steps: automatic detection of lobulation /microlobulation in a mammographic image and similar formalization of the other terms from the breast imaging lexicon. This study shows that fuzzy logic can be an effective tool in dealing with this kind of problems.

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