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POWER RESTORATION IN EMERGENCY SITUATIONS*

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Abstract: This paper addresses a problem of assigning electric power repair crews and depots to various locations (cells) in a damaged area during emergency situations such as natural disasters. The problem takes into account the damage levels and consequent demand for various resources at different cells in the area, along with the capacity restrictions of the depots. Two mixed integer quadratic programming models–one for single resource and another for multiple resources allocation are presented in the paper. The objective is to locate the depots, and assigning the crews and other resources to depots to various cells, at a minimum cost. The problem is solved optimally for various dimensions of the problem (e.g., number of cells in the region, number of depots considered and the number of resources) for a limited instance.

Key Words: *Power restoration, routing, allocation of resources, and mixed integer programming problem.*

1. INTRODUCTION

The problem of assigning electric power repair depots to various locations in a damaged area assumes critical importance during emergency situations such as natural disasters. This paper addresses this issue taking into account the damage levels and the consequent demand for various resources among different cells in the area, along with capacity limitations of the depots. While considerable amount of literature exists in the area of location research, it may be mentioned that the issue of simultaneous location of depots with varied capacities, and their assignments to customers with different demands has not been addressed. For the purpose of power restoration, a region may be divided into a number of cells. Each cell is affected by the disaster differently, and the damage levels may be assessed to ascertain the amount of resources required for power restoration in the cell. In a similar vein, the available depots may also differ in terms of their capacities with respect to various resources. The problem, then, is to determine the location of the available depots and also, to ascertain the amount of resource transported from different depots to various cells, so as to minimize the total transportation cost. It is implicitly assumed that this objective also enables restoration of power in minimal time. Two mixed integer quadratic programming models for the cases of single and multiple resources are presented (Batta and Mannur 1990, and Beasley 1993). An illustrative example is provided for a case with multiple resources.

2. RESOURCE ALLOCATION MODELS

As mentioned earlier, the demand arising at various cells is dependent on the level of damage, which may be different for each cell. This, coupled with the fact that all the depots may not be identical in terms of their capacities, leads to the formulation of the depot allocation problem as a mixed integer quadratic programming problem. Mathematical models of the depot allocation and resource transport problem are developed here. First we present a mixed integer quadratic programming model, considering a single resource requirement. Second, a mixed integer quadratic programming model is developed for the case of multiple resources (Eaton *et al.* 1985, and Toregas and Revelle 1992). Before proceeding with the model development, the notation followed may be presented as follows:

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Notation:

- i: an index for the depot ($i = 1, 2, ..., N$).
- j: an index for the resource type ($j = 1, 2, ..., m$).
- k: source cell ($k = 1, 2, ..., n$).
- h: target cell ($h = 1, 2, ..., n$).
- c_j = Transportation cost (c for single) per unit of resource j per unit distance.
- C_i = Capacity, in terms of the resource availability, of depot i.
- C_{ij} = Capacity of depot i in terms of availability of resource j.
- D_{jh} = Number of units of resource j demanded at cell h.
- x_{ijh} = Number of units of the resource j transported from depot i to cell h.
- y_{ik} = 1, if depot i is located in cell k, and 0 otherwise.
- Δ_{kh} = Distance between cells k and h (based on the given inter-cell distance matrix).

2.1 Single resource

First, one of the constraints considered for the problem is the demand requirement for each cell. In other words, the total number of units of resource supplied by different depots to a given cell h, given by $\sum x_{ih}$, should be equal to the cell demand, D_h. Second, the capacity of a depot limits the maximum number of units of resource transported from it to any cell. Third, each depot may be located at only one cell, and further, each cell may contain at the most one depot. For computing the transportation cost from cell k to cell h, one has to consider the amount of resource transported from cell k to cell h. We know that the number of units supplied by depot i to cell h is x_{ih}. These units will, in fact, be transported from cell k to cell h, if the depot i is located at cell k. Hence, mathematically, the number of units transported from cell k to cell h may be expressed by the quantity ($x_{ih}y_{ik}$) because y equals 1 if the depot i is located at cell k. If the distance between the cells k and h is Δ_{kh}, and transportation cost per unit resource per unit distance is c, the cost of transporting $x_{ih}y_{ik}$ from cell k to cell h would be given by ($c\Delta_{kh}x_{ih}y_{ik}$). Thus, the depot allocation model may now be stated as a mixed-integer quadratic programming problem as follows:

$$\text{Minimize } Z = \sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{h=1}^{n} c\Delta_{kh}x_{ih}y_{ik} \quad (1)$$

$$\text{Subject to } \sum_{i=1}^{N} x_{ih} = D_h, \quad \text{for } h = 1, 2, ..., n. \quad (1a)$$

$$x_{ih} \le C_i, \quad \text{for } i = 1, 2, ..., N \text{ and } h = 1, 2, ..., n. \quad (1b)$$

$$\sum_{k=1}^{n} y_{ik} = 1, \quad \text{for } i = 1, 2, ..., N. \quad (1c)$$

$$\sum_{i=1}^{N} y_{ik} \le 1, \quad \text{for } k = 1, 2, ..., n. \quad (1d)$$

$$x_{ik} \ge 0, \; y_{ik} = \{0,1\}, \quad \text{for } i = 1, 2, ..., N; \text{ and } k = 1, 2, ..., n. \quad (1e)$$

This formulation allows for differences in the capacity (resource availability) of the depots, and also enables assessment of damage in a particular cell in terms of the demand for the resource. While a single type of resource is considered here, the above model can easily be extended to take into account different types of resources, say, workers, materials, etc., which is dealt with in the following section.

2.2 Multiple resources

In the previous model, a single type of resource is considered in modeling the power restoration system. While this may be useful in some situations often multiple resources may be required for the repair of cells. Besides, different depots could have different capacities with respect to the different types of resource. These factors are considered in the model presented here. The number of units of resource j transported from cell k to cell h is given by ($x_{ijh}\, y_{ik}$), since x_{ijh} units supplied by depot i to cell h will be transported from cell k, if the depot i is located at cell k. Now, the cost of transportation for this is given by ($c_j\, \Delta_{kh}x_{ijh}y_{ik}$). The constraints for this problem are similar to those for the single resource case, albeit, with a modification considering the different resources in the demand and capacity constraints. The depot allocation problem for the case of multiple resource types may now be formulated as a mixed integer-quadratic programming model as follows:

$$\text{Minimize} \quad Z = \sum_{j=1}^{m} c_j \sum_{k=1}^{n} \sum_{h=1}^{n} \Delta_{kh} \sum_{i=1}^{N} x_{ijh} y_{ik} \tag{2}$$

Subject to equations (1c), (1d) and

$$\sum_{i=1}^{N} x_{ijh} = D_{jh}, \quad \text{for } j=1,\dots,m; \text{ and } h=1,\dots,n. \tag{2a}$$

$$x_{ijh} \le C_{ij}, \quad \text{for } i = 1, \dots, N; j = 1, \dots, m, \text{ and } h = 1, \dots, n. \tag{2b}$$

$$x_{ijh} \ge 0,\ y_{ik} = \{0,1\} \quad \text{for } i = 1, 2, \dots, N; j=1,\dots,m \text{ and } k = 1, 2, \dots, n. \tag{2c}$$

3. SOLUTION TO AN INSTANCE

The above model will be illustrated with the use of an example. For this purpose, three types of resources are considered, namely, workers, equipment type I, and equipment type II. Further, two depots are assumed to be available with their resource capacities as given in Table 1. The affected region is hypothetically divided into five cells. The cells, numbered 1 through 5, have the resource demand as given in Table 2. It may be mentioned here that the demand arising from a cell for different resource types is to be based on its assessed damage level. The transport cost per unit of resource transported per unit distance is given in Table 3. The inter-cell distance matrix is given in Table 4.

Input Data:

Table 1: Available capacity (in units) of the depots.

Depot	Capacity		
	Personnel	Equipment I	Equipment II
A	8	4	-
B	9	7	13

Table 2: Cell-wise demand (in respective units) for the resources.

Cell	Demand		
	Personnel	Equipment I	Equipment II
1	4	7	9
2	8	4	3
3	9	5	7
4	5	2	5
5	2	8	8

Table 3: Transportation Cost (dollars/unit/mile).

Resource	Cost
Personnel	10
Equipment I	4
Equipment II	6

Table 4: Inter-cell Distance Matrix (in miles)

Cell	Distance				
i \ j	1	2	3	4	5
1	0	5	2	3	8
2	5	0	6	4	7
3	2	6	0	1	9
4	3	4	1	0	10
5	8	7	9	10	0

Output Data:

The MIP model has been solved with these data using the GAMS software package. The minimum cost achieved is $1,268. The solution output has two components: the depot location (Table 5), and the units of resource transported from each depot to different cells for each of the resource types (Table 6).

Table 5: Depot Location.

Depot	Cell
A	2
B	3

Table 6(a): Number of Personnel Transported.

Depot	Cell				
	1	2	3	4	5
A	-	8	-	-	2
B	4	-	9	5	-

Table 6(b): Number of Units of Equipment I Transported.

Depot	Cell				
	1	2	3	4	5
A	-	4	-	-	4
B	7	-	5	2	4

Table 6(c): Number of Units of Equipment II Transported.

Depot	Cell				
	1	2	3	4	5
B	9	3	7	5	8

4. CONCLUDING REMARKS

Both models presented in this paper necessitate assessment of demand arising out of different cells for various types of resources based on the damage occurred in each cell. The example merely presents a sample solution to illustrate the models. Thus, the solution is limited to a small instance of the problem, and solutions to large problem instance will be obviously prohibitive due to its computational intractability. So a heuristic may be devised for an alternative approach to solve problems with large dimensions (i.e, number of depots considered, the number of types of resource required, and the number of cells in the affected region).

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