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A MODEL FOR ADDING NEW DEPOTS FOR EMERGENCY POWER RESTORATION

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ABSTRACT

The purpose of this paper is to develop a depot location model to manage the resources needed for the power restoration efficiently and economically in an area affected by a disaster. The problem adds new depot(s) to an area where some depots already exist. This decision-making trades off whether or not it is necessary to establish new depots. A mathematical model for both cases are developed on an incremental basis as the problems become complex and new scenarios become apparent in capturing the more general aspect of the problem. An optimal solution strategy is presented following each problem. The results show that the optimal model can be used in power restoration management.

Key Words: *Power restoration, and depot location.*

1. INTRODUCTION

The depot location problem addressed in this research arises from restoration of electric power after a disaster. For example, on August 26, 1992, hurricane 'Andrew' struck Louisiana and caused considerable loss of life and property. It knocked out power, blew down trees, closed highways and bridges and created general havoc throughout the region. Many power failures were the result of trees being toppled onto overhead power lines. In many areas, power could not be restored for more than 2 weeks. Industries, businesses, schools, and other facilities were closed. The disaster also caused indirect losses from taxes and revenue lost due to the delay in power restoration. The primary need after that hurricane was to restore electricity as early as possible. Unfortunately, the power utility companies lacked the resources to deal with such a sudden need for a large number of emergency repair crews and vehicles. As a result, they requested help to facilitate the restoration process. Major power companies sent in extra crews from other states and other areas in Louisiana.

Therefore, the main problem of the power companies was to manage these arriving resources in an efficient and economic way so as to restore normalcy. An efficient way to do this was to set up or locate the depots in the area first, and then prepare for restoration. Each depot was equipped with the necessary repair equipment, vehicles, crews, and other resources. Thus, in this logistic problem, determining optimal number of depots, optimal locations of depots, and optimal number of repair crews and/or vehicles are of significance to the efficient restoration of operations. The objective of this research is to present a depot location model for whether or not to add additional depot(s) to restore power after a disaster.

ADDING NEW DEPOTS TO AN AREA

In this model, the depots may be different from each other, that is, the types of the resource and the amount of each type of resources at each depot may be different. The model constitutes a major difference from the classical location models in that the model considers different depots, multiple resources, no limitation of cell for depot location, and no limitation in the amount of the resources transported from different depots to various cells, simultaneously. This problem considers the condition that, in a restoration area where some depots are already located, after a disaster, an emergency area is formed. The model answers the questions: Does it need additional depot(s) in the emergency area? how many? and where?

NEW DEPOT MODEL AND SOLUTION

In this section, a depot location model is developed for general power restoration situations with emergency area identified as a separate area is described. The notation used in the models is:

Notation:

X_{kij} Quantity of resource k ($k=1, 2,..., m$) shipped from depot i ($i = 1,...,n$) to cell j ($j = 1,...,N$);

A_{ik} Capacity matrix for depot i and resource k;

P_{hkij} Amount of resources k transported from depot i located at cell h to cell j;

D_{jk} Demand of cell j for resource k;

C_k Unit transportation cost of resource k;

d_{ij} Distance between cells i and j;

δ_{ij} Binary $(0, 1)$ integer variables. It equals one if depot i is located at cell j or zero, otherwise.

Here an emergency area is identified after a disaster, where some depots are already located. To restore the power in this area (the whole area, including the affected and non-affected areas), the problem is whether or not it is necessary to locate new depot(s). If so, how many depots are to be allocated and where to locate them? The decisions can be made by comparing the total cost of the cases before and after adding. Two cases of restoration in this emergency area are studied here. The first one is the resource assigning problem which does not add any depot and demand, although changed, is still satisfied by the pre-existing depots. The second one is the problem that adds new depots. The models and solutions of them are described below.

Adding new depot(s)

Let H be the holding cost of existing depots and K the fixed cost to build new depot(s). Suppose in a restoration area, depots p_i $(i=1, 2,..., n_1)$ are already located at cells q_j $(j=1, 2,..., n_1)$. After the disaster happens, the emergency area is marked for ease of modeling purpose whether no additional depot is necessary to be added in the affected area or if the demands of all cells including those in emergency area are to be served by existing depots p_i $(i=1, 2,..., n_1)$ located at cells q_j $(j=1, 2,..., n_1)$.

Suppose it is planned to add n_2 new depot(s) to appropriate cells in the emergency area. The demand for the whole area can be serviced by both the existing depots and the new depots. For the existing depots, it remains a resource assignment problem. For the new depots, it is a location problem, i.e., first find the optimal depot locations, then transfer resources to the corresponding cells. Considering the fixed cost and holding cost, the power restoration problem can be formulated as follows:

(ND): *Min*

$$TC = \sum_{i=1}^{n_2} K_i + \sum_{i=1}^{n_1+n_2} H_i + \sum_{k=1}^{m} \sum_{i=1}^{n_1} \sum_{j=1}^{N} C_i d_{ij} X_{kij}$$
$$+ \sum_{h=1}^{N_1} \sum_{k=1}^{m} \sum_{i=1}^{n_2} \sum_{\substack{j=1 \\ \forall j \in \omega 1}}^{N_1} C_i d_{ij} P_{hkij}$$

Subject to

$$\sum_{j=1}^{N} X_{kij} \le A_{ik} \quad i=1, 2,..., n, k=1, 2,.., m$$

$$\sum_{i=1}^{n} \sum_{j=1}^{N} X_{kij} \ge D_{jk}, \ k=1, 2,..., m$$

$$P_{hkij} + M(1- N_{ij}) \ge X_{kij}, \quad i=1,...,n_1,; j=1,...,N_1; \ k=1,...,m; \ h=1,..., N_1$$

$$M = 2 * max\{X_{kij}\}$$

$$\sum_{j=1}^{N_1} N_{ij} = 1, \ i = 1, 2,..., n_1$$

$$\sum_{i=1}^{n_1} N_{ij} \le 1, \ j = 1, 2,..., N_1$$

$$X_{kij} \ge 0, P_{hkij} \ge 0, N_{ij} = (0,1), i=1,...,n_1,; j=1,...,N_1; \ k=1,...,m; \ h=1,..., N_1$$

In the objective function equation (2), the first and second terms are the fixed and holding cost, respectively. The third term is the shipping cost incurred by the existing depots. The last term is the shipping cost with respect to the new depot(s). To determine the optimal number of additional depot(s), two cases are considered: (a) the demand of the emergency area is larger than the capacity of existing depots, i.e., $D_{jk} > A_{ik}$, and (b) the demand is less than the capacity of existing depots, i.e., $D_{jk} < A_{ik}$. These cases are explained below.

(a) $D_{jk} > A_{ik}$

A new depot(s) needs to be added to the emergency area. The number of additional depot is determined by trial and comparison. Algorithm A produces the solutions to resolve this problem.

Algorithm A:

Step 1. One depot is added and the total cost in Problem ND is calculated and stored;

Step 2. Increase the depot number by one, i.e., depot number $n = n + 1$. Calculate the total cost in Problem ND and store it;

Step 3. Compare the total costs of two different depot numbers. If the total cost of added $n + 1$ depots is greater than that of added n depots, then stop. The decision is made that n depots are added. Otherwise, update the total cost of adding n depots by adding $n + 1$ depots. Go to Step 2.

(b) $D_{kj} < A_{ki}$

In this case, it may or may not be necessary to add new depot(s) to the emergency area. So first, the total cost of Problem RAP is calculated. Then the total cost of adding one depot is calculated in Problem ND. If the former is less than the latter, then stop and no new depot is necessary. Otherwise, it is necessary to add additional depot(s) to the emergency area. The optimal number of additional depots is determined in the same procedure as described in condition (a).

Optimal solution for adding new depot(s)

It is planned to add two new depots, *c* and *d* which have the capacities for each resource as shown in the capacity matrix $\boldsymbol{A}$. The unit cost matrix $\boldsymbol{C}$, and inter-cell distance matrix $\boldsymbol{d}$ remains the same. The fixed cost and holding cost of each depot are given in Table 1.

Example 1: The capacity matrix $\boldsymbol{A}$, unit cost matrix $\boldsymbol{C}$, demand matrix $\boldsymbol{D} = [D_{jk}]$, and inter-cell distance matrix $\boldsymbol{d} = [d_{ij}]$ are given as follows:

$$A = \begin{array}{c} \text{Depot} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} R1 & R2 & R3 \end{array} \\ \begin{bmatrix} 8 & 4 & 0 \\ 9 & 8 & 13 \\ 5 & 3 & 6 \\ 6 & 3 & 5 \end{bmatrix} \end{array} \qquad D = \begin{array}{c} \text{Cell} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} R1 & R2 & R3 \end{array} \\ \begin{bmatrix} 14 & 10 & 9 \\ 16 & 8 & 8 \\ 13 & 12 & 9 \\ 9 & 5 & 10 \\ 8 & 10 & 8 \end{bmatrix} \end{array}$$

$$C = \begin{array}{c} \text{Resource } k \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix} \end{array} \qquad d = \begin{array}{c} \text{Cell } i \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \text{Cell} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 5 & 2 & 3 & 8 \\ 5 & 0 & 6 & 4 & 7 \\ 2 & 6 & 0 & 1 & 9 \\ 3 & 4 & 1 & 0 & 10 \\ 8 & 7 & 9 & 10 & 0 \end{bmatrix} \end{array}$$

Table 1. The fixed cost and holding cost of each depot.

	Depot a	*Depot b*	*Depot c*	*Depot d*
Fixed cost, K ($)	0	0	950	950
Holding cost, H ($)	80	50	50	50

The condition of $D_{kj} > A_{ik}$ and of $D_{kj} < A_{ik}$ is verified before solving the problem. In this set of data, $D_{kj} < A_{ik}$, holds because the capacity of each depot is larger than the demand of the individual cells for every type of resource. In order to solve the problem, the total costs of three cases should be obtained: (a) No depot is added, the demand is supplied by existing depots *a* and *b*; (b) If one depot *c* is added; (c) If two depots, *c* and *b* are added. The total costs are then compared and a decision is made on how many additional depots are needed. Solutions for case (a) have already been obtained in Example 2. Following is the solutions for cases (b) and (c).

Case (b): Adding depot c: The depot *c* should be allocated at cell 5, which makes the minimum shipping cost $2,586. The units shipped (the values of variables) by three depots are ascertained by Problem ND.
Case (c): Adding two depots, c and d: The depot *c* should be allocated at cell 4, and depot *d* at cell 5, which makes the minimum shipping cost $1,760. The units shipped (the values of variables) by the four depots are ascertained by Problem ND.

Next we provide all the results on shipment, and known data into a table and to make decisions. Table 2 shows the cost of adding new depots. It can be observed from Table 2 that the total cost of case (a), $3,498, is the smallest among the three cases. So, a decision is made to use existing depots *a* and *b* to serve the demand, no additional depot needs to be added in.

Table 2. Costs of adding new depots

Condition	*Fixed cost*		*Holding cost*				*Ship*	*TC($)*	*Decision*
	c	*d*	*a*	*b*	*c*	*d*	*($)*		
(1)	-	-	80	50	-	-	3368	3498	Yes
(2)	950	-	80	50	50	-	2586	3716	No
(3)	950	950	80	50	50	50	1760	3890	No

(1) Using *a* and *b*, (2) Add 1 depot, (3) Add 2 depots.

Table 3. Costs of adding new depots

Condition	*Fixed cost*		*Holding cost*				*Ship*		*Decision*
	c	*d*	*a*	*b*	*c*	*d*	*($)*	*TC($)*	
(1)	-	-	80	50	-	-	3368	3498	No
(2)	750	-	80	50	50	-	2586	3516	No
(3)	750	750	80	50	50	50	1760	3490	Yes

(1) Using *a* and *b*, (2) Add 1 depot, (3) Add 2 depots.

If the fixed cost of new depots is $750 and other data remain the same (as shown in Table 3), the decision will be for case (c): adding two depots that amounts to $3,490, which is the smallest. This result is interesting because it is worth adding two depots rather than one.

CONCLUSIONS

The depot location model can simultaneously find the optimal locations for depots, and allocate the differing types of resources shipped from individual depot to each cell to meet the demand of cells. The model is aimed at minimizing transportation time and cost to make the power restoration efficient. For the restoration of emergency area, Problem ND provides the decision-makers with an evaluation tool to tradeoff adding or not adding additional depots and the number of depots. The depot location problem in this scenario is usually modeled as a mixed-integer quadratic programming (MIQP) which has difficulty in obtaining an optimal solution for large cases. Fortunately, it can be converted into an equivalent integer linear programming (ILP). For the ILP, the optimal solutions are obtained easily but the conversion results in a great number of additional variables and constraints, which still limit the software to solve small (about cell) problems.

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