* For the reference see also the last page.

A DEPOT LOCATION MODEL FOR ELECTRIC POWER RESTORATION

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ABSTRACT: The purpose of this research is to develop a depot location model to manage the resources needed for restoring power efficiently and economically to an area affected by a disaster. The problem simultaneously locates depots and determines the amounts of the resources shipped from the depots to each point of demand to satisfy the demands of the various locations in order to minimize the total transportation cost. A mathematical model is developed on an incremental basis as the problems become complex. The results show that the optimal model can be used in power restoration management. For a large size problem, a two-phase heuristic is developed.

Keywords: *Power restoration, depot location, linear integer programming.*

1. INTRODUCTION

The depot location problem addressed here arises from restoration of electric power after a disaster. For example, on August 26, 1992, hurricane 'Andrew' struck Louisiana and caused considerable loss of life and property. In many areas, power could not be restored for more than 2 weeks. Industries, businesses, schools, and other facilities of public interests were closed. The disaster also caused indirect losses because of the delay in power restoration. The primary need after that hurricane was to restore electricity as early as possible. Unfortunately, the utility companies lacked the resources to deal with such a sudden need for a large number of emergency repair crews and vehicles. An efficient way to do so was to set up or locate the depots in the area first and then prepare for restoration. Each depot was equipped with the necessary repair equipment, vehicles, crews, and other resources. Thus, in this logistic problem, determining optimal number of depots, optimal locations of depots, and optimal number of repair crews and/or vehicles are of significance. The objective of this research is to present a depot location model for power restoration problem.

2. PROBLEM IDENTIFICATION

The general problem is to simultaneously locate the depots and determine the resources shipped from the depots to each cell to satisfy the demands of the various cells, while minimizing the total transportation cost between depots and cells. Fig.1 shows this kind of problem in which there are two depots, A and B, and 6 cells. Either depot may be assigned to any one of these 6 cells, but the requirement is that, in each cell only one depot can be located. The research question is determine the best route, that is, how to locate these two depots in the area in such a way that satisfies the demand of cells and minimizes overall transportation cost? The model constitutes a major difference from the classical location models in that the model considers simultaneously different depots (with no restriction on cells for depot location), multiple resources and the amount of the resources transported from different depots to various cells.

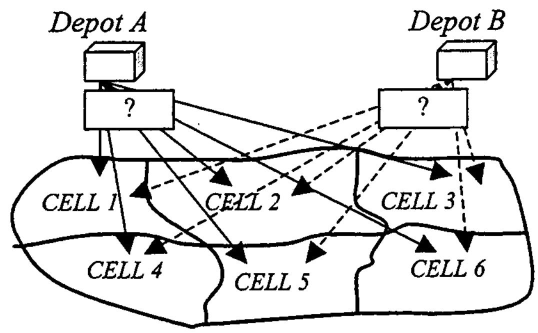

Fig. 1. A depot-location problem.

3. MODEL FORMULATION AND OPTIMAL SOLUTIONS

Let X_{kij} be the quantity of resources k ($k=1, 2, \ldots, m$) shipped from depot i ($i = 1, 2, \ldots, n$) to cell j ($j = 1, 2, \ldots, N$) and δ_{ij}. The binary $(0, 1)$ integer variables. It equals one if depot i is located at cell j or zero, otherwise.

3.1 The general depot location problem

If depot i ($i = 1, 2, \ldots, n$) is located at cell j ($j = 1, 2, \ldots, N$), the resource k ($k=1,2,\ldots, m$) transported from cell i to cell j is given by the product $\delta_{ij} X_{kij}$. The cost of transportation for the amount of resource k can be written as $C_k d_{ij} \delta_{ij} X_{kij}$. Therefore, the total intercell transportation cost is given by:

Problem GP: $$Min \;\; TC = \sum_{k=1}^{m} C_k \sum_{i=1}^{n} \sum_{j=1}^{N} d_{ij} \delta_{ij} X_{kij} \tag{1}$$

Subject to

$$\sum_{j=1}^{N} X_{kij} \le A_{ik} \qquad i=1, 2, \ldots, n,\; k=1, 2, \ldots, m \tag{1a}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{N} X_{kij} \ge D_{jk} \qquad k=1,2,\ldots,m \tag{1b}$$

$$\sum_{j=1}^{N} \delta_{ij} = 1 \qquad i= 1, 2, \ldots, n \tag{1c}$$

$$\sum_{i=1}^{n} \delta_{ij} \le 1 \qquad j=1,2,\ldots,N \tag{1d}$$

$$X_{kij} \ge 0 \qquad i=1, 2, \ldots, n,\; j=1, 2, \ldots, N,\; k=1, 2, \ldots, m \tag{1e}$$

$$\delta_{ij} = (0,1) \qquad i=1, 2, \ldots, n, j=1, 2, \ldots, N \tag{1f}$$

The allocation model is a mixed-integer quadratic program (MIQP) and hence, it is difficult to solve it optimally. In order to obtain the optimal solutions for relatively large cases, it should be converted into an integer linear programming model.

3.1.1. An equivalent integer linear model

To obtain an equivalent integer linear model, a four dimensional variable P_{hkij} is introduced. Let $P_{hkij} = \delta_{ij} X_{kij}$. The equivalence procedure takes place in an additional constraint called "variable switching constraint". An integer linear program (ILP) can then be written as follows:

Problem ILP: $$Min \quad TC = \sum_{h=1}^{N} \sum_{k=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{N} C_i d_{ij} P_{hkij} \tag{2}$$

Subject to (1a-1f) and the variable switching constraints:

$$P_{hkij} + M(1-\delta_{ij}) \geq X_{kij}, \quad i=1, \ldots, n,\ j=1, \ldots, N;\ k=1, \ldots, m;\ h=1, \ldots, N \tag{2c}$$
$$P_{hkij} \geq 0, \quad i=1, \ldots, n,\ j=1, \ldots, N;\ k=1, \ldots, m;\ h=1, \ldots, N \tag{2g}$$

Here, M is a big number. The advantage of this model is that it is linear and it is easier to solve. The disadvantage lies in the complexity of the constraints and variables. As compared to the problem GP, the additional variable number is Nmn^2, and the additional constraint number is $2Nmn^2$.

3.1.2. Optimal solution for the depot location model (Problem ILP)

Assume an ILP depot location model with a 2 depot, 5 cell, and 3 types of resource problem. Suppose two depots, a and b, need to be allocated in the area. There are, three types of resources(R1, R2, R3) stored at each depot. The capacity matrix $\boldsymbol{A}$, demand matrix $\boldsymbol{D} = [D_{jk}]$, unit cost matrix $\boldsymbol{C}$, and inter-cell distance matrix $\boldsymbol{d} = [d_{ij}]$ are given below:

$$A = \begin{array}{c} \text{Depot } i \\ a \\ b \end{array} \begin{array}{c} \begin{array}{ccc} R1 & R2 & R3 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 5 & 3 & 1 \end{bmatrix} \end{array} \qquad C = \begin{array}{c} R \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 15 \\ 4 \\ 11 \end{bmatrix} \end{array} \qquad D = \begin{array}{c} \text{Cell } i \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} R1 & R2 & R3 \end{array} \\ \begin{bmatrix} 5 & 2 & 3 \\ 2 & 4 & 3 \\ 5 & 4 & 3 \\ 1 & 4 & 3 \\ 4 & 4 & 0 \end{bmatrix} \end{array} \qquad d = \begin{array}{c} \\ \text{Cell } i \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \text{Cell } j \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 9 & 9 & 20 & 2 \\ 9 & 0 & 0 & 25 & 70 \\ 46 & 4 & 4 & 20 & 69 \\ 20 & 25 & 25 & 0 & 11 \\ 2 & 70 & 70 & 11 & 0 \end{bmatrix} \end{array}$$

The solution yielded the values of variables (the units of resources to be shipped) as shown in Table 1.

Table 1. The units shipped from depots

Resource	*Cell* / *Depot*	1	2	3	4	5
Units of R1 Transported	*a*	0	8	0	0	2
	b	4	0	9	5	0
	Total	4	8	9	5	2
Units of R2 Transported	*a*	0	4	0	0	4
	b	7	0	5	2	4
	Total	7	4	5	2	8
Units of R3 Transported	*a*	0	0	0	0	0
	b	9	3	7	5	8
	Total	9	3	7	5	8

The solution indicates that depot a is located at cell 2 and depot b at cell 3. The units are shipped from these two cells which make the optimal total cost $7,121.

4. THE TWO-PHASE HEURISTIC

Although the optimal solution of an integer linear programming problem can be obtained for a relatively large matrix, it is still small with respect to a realistic problem. For a large size problem, a heuristic helps the researcher obtain an approximate optimal solution. In this research, a two-phase heuristic for the depot location is developed. Phase 1 determines the depot location, and Phase 2 determines the amount of shipment from the depots. To complete the two-phase heuristic, two matrices should be introduced: the first one is the total cost matrix ***TCC,*** and the second one is the shipment cost matrix ***TCD.*** They are given by:

$$TCC(i,j) = \begin{cases} \sum_{h=1}^{N}\sum_{k=1}^{m} d(j,h)A(i,k)C(k) & \text{if } D(h,k) \leq A(i,k) \\ \sum_{h=1}^{N}\sum_{k=1}^{m} d(j,h)A(i,k)C(k) & \text{if } D(h,k) \leq A(i,k) \end{cases} \tag{3}$$

$$TCD(i,j) = \sum_{k=1}^{m} d(i,j)D(j,k)C(k) \tag{4}$$

4.1. Algorithm 1: *Phase 1: Depot allocation*

Step 1: Calculate the total cost $TCC(i, j)$;
Step 2: Find n smallest values of y_{ik} ($i = 1, 2, ..., n$, $k = 1, 2, ..., n$) in each row of ***TCC***;
Step 3: Combine the values in each column; $X_{(1,k1),(2,k2),\dots,(n,kn)} = y_{1k1} + y_{1k2} + \dots + y_{nkn}$
Step 4: Find the smallest value of $X_{(1,k1),(2,k2),\dots,(n,kn)}$ whose indexes $k1, k2, \dots, kn$ of are not the same.
Step 5: Assign depot *1* to cell k_1, depot *2* to cell k_2, ..., depot n to cell k_n.

4.3.1. Algorithm 2: *Phase 2: Ascertaining resources*

Step 1: Calculate ***TCD***;
Step 2: Arrange the $TCD(i, j)$, $i = 1, 2, ..., N$, $j = 1, 2, ..., N$, in increasing order;
Step 3: Ascertain the resources transported to each cell from different depots;
Step 4: Repeat Step 3 and 4 until $j = N$.

4.2. Illustration of Two-Phase Heuristic

To illustrate the Two-Phase heuristic, the same example is used again. The total cost matrix ***TCC*** is computed and is given in Table 2.

Table 2. A total cost matrix $TCC(i, j)$.

Depot / *Cell*	*1*	*2*	*3*	*4*	*5*
a	1958	1268	2096	1734	3952
b	5889	7228	10264	5957	11078

Table 3. The shipment cost $TCD(i, j)$

$$TCD = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 711 & 5704 & 1280 & 152 \\ 1044 & 0 & 496 & 1600 & 5320 \\ 5336 & 316 & 0 & 1280 & 5244 \\ 2320 & 1975 & 2480 & 0 & 836 \\ 232 & 5830 & 8556 & 704 & 0 \end{bmatrix} \end{array}$$

By following the algorithm 1.1, the depot allocation is given as: Depot *a* is assigned to cell 2 and depot *b* is assigned to cell 1. The next step is to determine the amount of resources that should be shipped from the appropriate depots. This is done by Phase 2. In Phase 2, the shipment cost matrix ***TCD*** is calculated and shown in Table 3. Because depot *a* and *b* are assigned at cell 2 and 1, respectively, in each column, only the elements of these two rows (row 2 and row 1, as shaded in table) need to be compared. After ascertaining, the values of variables are the same as those obtained in ILP Problem which are shown in Table 1. The total cost obtained by Two-Phase heuristic is given by: TC = $7,121. The performance of the heuristic is satisfactory in this small size example. The algorithm is based on total cost matrix ***TCC*** allocating to the depots and based on the shipment cost matrix ***TCD*** assigning the units from the depots to cells.

5. CONCLUSIONS

The depot location problem in this scenario is usually modeled as a mixed-integer quadratic programming (MIQP) difficult to obtain an optimal solution for large cases. Fortunately, it can be converted into an equivalent integer linear programming (ILP). For the ILP, the optimal solutions are obtained easily but the conversion results in a great number of additional variables and constraints, which still limits the software in solving large problems. A heuristic is considered effective to provide an approximate solution. The results show that the two-phase heuristic developed in this research works well for the depot location problem.

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