* For the reference see the last page.

Decision Making in Engineering and Business: A Cost-Effective Approach

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Abstract

A critical issue in all real life decision-making problems, is the effective estimation of the pertinent data. Often data are difficult to be estimated with accuracy, or too dynamic to allow for a timely estimation. This is the case with most of the environmental, resource management, maintenance, economic, agricultural, and manufacturing problems everywhere, especially in the United States. If the input data are not correct, then the decision making process may cause a problem more severe than the one which it intended to solve. In this paper we propose an approach for a cost-effective estimation of the pertinent data. This approach is based on a sequence of successive sensitivity analyses.

1. Introduction

One of the most common problems in many engineering and business applications is how to evaluate a set of alternatives in terms of a set of decision criteria. For instance, one may need to upgrade a computer system. There is a number of different configurations available to choose from. The different systems are the alternatives. A decision should also consider the cost, performance characteristics (i.e., CPU speed, memory capacity, RAM, etc.), availability of software, maintenance, expendability, etc. These may be some of the decision criteria for this problem.

In the above problem we are interested in determining the <u>best alternative</u> (i.e., computer system). In some other situations, however, one may be interested in determining the relative importance of all the alternatives under consideration. For instance, if one is interested in funding a set of competing projects (which are the alternatives), then the relative importance of these projects is required (so the budget can be distributed proportionally to their relative importance).

Multi-criteria decision-making (or MCDM) plays a critical role in many real life problems. It is not an exaggeration to argue that almost any local or federal government, industry, or business activity involves, in one way or the other, the evaluation of a set of alternatives in terms of a set of decision criteria. Very often these criteria are conflicting with each other. Even more often the pertinent data are very expensive to collect. **<u>The main difficulty in real life applications of MCDM is not how to process the numerical data, but instead, how to effectively estimate them.</u>**

It should not be a surprise that many failures of MCDM recommendations were attributed to the bad quality of the input data. Since there is **<u>always a limited budget</u>** for solving a MCDM problem, then there is an urgent need for developing a methodology based on a sensitivity analysis for solving these types of problems.

As practical application of the above problem consider a simulation study which aims at investigating the potential of a number of alternatives. These alternatives could be different plant configurations with different hardware etc. Suppose that the management needs to make a decision by a certain deadline. In a case like this one, the budget is the limited CPU time available to the decision analysts. Therefore, the main goal is to allocate the available CPU time in such a way that the best alternative will be selected at the end with an acceptable confidence level. As a second application consider the case of a car manufacturer who wishes to execute some crash tests in order to evaluate alternative car designs. Apparently, the decision analysts would like to determine the best design by crashing a very small number of cars. Other practical applications can be found in many environmental problems, policy analysis, economic analysis, just to name a few.

Applications of deterministic MCDM are numerous and diverse. MCDM methodologies have found many applications in environmental related problems. In [Janssen, 1992] a multi-objective decision support methodology is presented for environmental management with some emphasis on sensitivity analysis. Other areas of application of deterministic MCDM include manufacturing problems (such as the ones discussed in [Putrus, 1990]), social problems (such as the ones discussed in [Hwang and Yoon, 1980] and [Hwang and Lin, 1987]), and planning problems (as the ones discussed in [Saaty, 1980], and [Golden, Wasil and Harker, 1989]).

2. Structure of the Decision Problem Under Consideration

The structure of the typical decision problem considered in this paper consists of a number, say M, of alternatives and a number, say N, of decision criteria. Each alternative can be evaluated in terms of the decision criteria and the relative importance (or weight) of each criterion can be estimated as well.

Let a_{ij} (i=1,2,3,...,M, and N=1,2,3,...,N) denote the **performance value** of the i-th alternative (i.e., A_i) in terms of the j-th criterion (i.e., C_j). Also denote as W_j the **weight** of the criterion C_j. Then, the core of the typical MCDM problem can be represented by the following **decision matrix**:

	Criterion				
	C_1	C_2	C_3	...	C_N
Alt.	W_1	W_2	W_3	...	W_N
A_1	a_{11}	a_{12}	a_{13}	...	a_{1N}
A_2	a_{21}	a_{22}	a_{23}	...	a_{2N}
A_3	a_{31}	a_{32}	a_{33}	...	a_{3N}
.	.	.	.	.	.
.	.	.	.	.	.
A_M	a_{M1}	a_{M2}	a_{M3}	...	a_{MN}

Given the above decision matrix, the decision problem considered in this study is how to determine which is the best alternative. An alternative problem is to determine the relative significance of the M alternatives when they are examined in terms of the N decision criteria combined. For instance, if the decision problem is to select the best project to be funded, one is only interested in identifying the best candidate project. However, in the problem of allocating a budget among a number of competing projects, one may be interested in identifying the relative importance of each project, so that the budget can be distributed proportionally to the significance of each project.

In a simple MCDM situation, all the criteria are expressed in terms of the same unit (e.g., dollars). However, in many real life MCDM problems different criteria may be expressed in different units. Examples of such units include dollar figures, political impact, environmental impact, etc. It is this issue of multiple dimensions which makes the typical MCDM problem to be a complex one.

The main problem investigated in this paper is best illustrated in the following numerical example. Suppose that a MCDM problem involves four alternatives and four decision criteria. Assume that all the criteria are expressed in the **same unit**. Therefore, the MCDM method to be used in this particular illustrative example is the widely used **Weighted Sum Model (or WSM)**. According to this model the final priority (denoted as P_i) of each alternative is calculated by the following formula:

$$P_i = \sum_{j=1}^{N} a_{ij} W_j, \quad for \; i = 1,2,3, \ldots, M. \quad (1)$$

Then, the alternatives are ranked in terms of the previous final priority values. When the criteria are described in terms of different units, other methods (such as the Weighted Product Method [Hwang and Yoon, 1980], or the AHP, just to name a few) are more appropriate.

3. Problem Description

Next, assume that a **limited budget** is available to the decision maker (DM) (for simplicity we consider that there is only one decision maker) to be used for determining the values of the pertinent data (i.e., the a_{ij} and W_i values, for i,j = 1,2,3,4). At this point suppose that the decision maker has spent a portion of the original budget (say, 50%) to gain a first estimate of the data for this problem. To help fix ideas, suppose that the DM has estimated that the four criteria are associated with the following weights of importance:

$W_1 = 0.3277$, $W_2 = 0.3058$,
$W_3 = 0.2877$, and $W_4 = 0.0790$.

Furthermore, suppose that the DM has also estimated that the performance of each alternative in terms of the four decision criteria is as depicted in the following matrix:

	Criterion			
	C_1	C_2	C_3	C_4
Alternative				
A_1	0.5878	0.7799	0.5843	0.2822
A_2	0.4118	0.9309	0.2652	0.9234
A_3	0.8585	0.6656	0.1805	0.0844
A_4	0.0457	0.3154	0.4812	0.1785

It is very important to recall here that the above data were derived by spending only a part of the original budget. Therefore, the above data may or may not be accurate estimates of the actual data for this problem. That is, if more money is allocated in determining a particular item of data (say for instance, element $a_{2,3}$), then the more accurately that particularly piece of data will be estimated. For simplicity assume that each of the above data has been estimated with the same accuracy. That is, each element (i.e., a_{ij} or W_j value) is associated with exactly the **same standard deviation**. Also note that when formula (1) is applied, then the final preferences and ranking of the alternatives are derived to be as follows:

TABLE 1 Current Preferences

Alternative	Preference P_i	Ranking
A_1	0.6214	1*
A_2	0.5688	2
A_3	0.5434	3
A_4	0.2639	4

Note: "*" *indicates the most preferred alternative (in the maximization case).*

Since the DM still has a portion of the original budget not committed, he/she desires to spent it in a way which would assure that this MCDM problem is solved as accurately as possible. ***The problem examined here is how to determine what is the best way to spent the rest of the budget such that the decision maker can solve a MCDM problem as accurately as possible given a limited budget.*** This problem is not a trivial one for the following reasons. In the current numerical example criterion W_1 has the highest weight (e.g., 0.327666), therefore this criterion is the **most important** one.

Suppose the DM wishes to spent some of the remaining budget in increasing the accuracy of the weight of the **most critical** (not necessarily the most important) criterion. Therefore, should he/she spent more money in increasing the accuracy of the estimation of W_1?

4. Proposed Methodology

In order to answer the previous question observe the following. When the weight of criterion C_1 is decreased by 0.2991 (and the weights are normalized again), then alternatives A_1 and A_2 become equally preferable (i.e., now we have: $P_1 = P_2 = 0.6357$). If the weight of criterion C_1 is decreased by amounts less than 0.2991, then the ranking of the four alternatives remains as originally. This critical value by which the weight W_1 has to be decreased such that a change in the current ranking will occur, was determined according to the formula in a theorem, which is presented later in section 6. For this particular example, it can be found (as it is explained in more detail later in section 6) that the corresponding critical changes of the weights of each criterion are as in table 2 (shown in the next page).

For instance, in table 2 if the current weight of criterion C_3 is decreased by 0.19322, then alternatives A_1 and A_3 will change ranking. That is, alternative A_3 will become more preferred than alternative A_1. A similar interpretation holds for the remaining entries. The notation *"N/F"* (for Non Feasible) indicates that no matter how the corresponding weight changes (as long as it does not becomes negative), the pair of alternatives shown on the first column will never change ranking.

Table 2 demonstrates that the criterion for which the smallest change in its current weight will cause a change on the current ranking of the four alternatives is criterion C_4. This is true because the minimum critical change corresponds to quantity 0.030248 which is under criterion C_4. When weight W_4 is modified by an amount greater than 0.030248, then alternative A_3 becomes more preferred (in the maximization case) than alternative A_2.

To answer the question which criterion should be determined with higher accuracy, table 2 indicates that it should be criterion C_4 and not C_1. **That is, criterion C_4 is the most critical one, while criterion C_1 is the most important one.** This realization was possible only after the above **sensitivity analysis** on the weights of the four criteria. Suppose that the DM is interested in determining only the best alternative. Then, he/she should focus only on the first three rows of table 2. This is true because the best alternative is examined only on the first three rows. In this case, the minimum critical change is -0.0821, which, by coincidence, corresponds to criterion C_4. That is, when the focus of the MCDM problem is to determine the best alternative (only), then the most critical criterion is criterion C_4. However, in general this may not always be the case.

The above brief sensitivity analysis reveals that the DM should spent part of the remaining budget to determine with higher accuracy the weight of the most critical criterion. If the weight of the most critical criterion has been determined with high enough accuracy (i.e., the corresponding standard deviation is very low), then the DM should spent more money in determining the next most critical weight and so on. Obviously, when the values a_{ij} are also considered, then a similar development is possible.

In general, we also assume that there is a **setup cost** which occurs when the decision maker gathers a set of new estimates. Therefore, we would like first to spent a part of the initial budget to get a first estimate of the pertinent data elements (i.e., the a_{ij}, and W_j values). Next, the DM spends a setup cost and estimates with higher accuracy the data elements which are the most critical ones (in the context of the previous sensitivity analysis consideration). The corresponding standard deviations should also be considered. That is, if an element is associated with a very low standard deviation, then even if it is very critical, we should spent more money estimating another less critical element (which is associated with a higher standard deviation). Clearly, a sensitivity analysis is needed at each step to determine what is the best distribution of the budget.

If the setup cost is equal to zero (or negligible), then we would like to proceed in many steps. That is, the estimation of the data, given a limited budget, should follow a **stepwise approach** with many steps. In this way the DM can be sure that the budget will be spent in estimating with higher accuracy only the most critical data (and also the ones with the highest variance). On the other hand, if the setup cost is very high, then only very few steps should be taken. However, if we take only a few steps, then it is possible to end up with the wrong understanding of the pertinent data (since a wrong initial estimation may mislead the process). Therefore, a goal here is to determine the optimal (or near optimal) number of such steps and portions of the budget to be spent at each step. Another related issue is how much of the original budget should be spent for the first estimation of the data.

5. A Sensitivity Analysis Approach

From the previous considerations and discussions it follows that **the first major problem** to be examined in a situation like the previous one is to identify which data (i.e, a_{ij} and W_j values) are the most critical ones. The most critical data are the ones for which the smallest change will alter the current ranking of the alternatives. Since data in real life applications of MCDM are easily changeable over time, or are difficult to be determined with high accuracy, it is of paramount importance to know which data are the most critical ones. By knowing which data are the most critical, the decision maker(s) can determine these data with higher accuracy.

TABLE 2

Critical amounts of criteria weight changes which cause alteration on the current ranking of the four alternatives.

Pair of Alternatives	Criterion			
	C_1	C_2	C_3	C_4
A_1 - A_2	0.2991	-0.34866	0.165	-0.0821
A_1 - A_3	-0.2883	N/F	0.19322	N/F
A_1 - A_4	N/F	N/F	N/F	N/F
A_2 - A_3	-0.05682	0.09567	N/F	0.030248
A_2 - A_4	N/F	N/F	N/F	N/F
A_3 - A_4	N/F	N/F	N/F	N/F

Otherwise, the decision maker(s) may **waste** a lot of money in determining data which are not so important in deriving the correct decision. The main challenge facing real life applications of MCDM today, is not how to process the numerical data, but instead, how to identify those parts of the problem which are the most critical in deriving the final (and correct) decision. Under a limited budget, it is not efficient to estimate with the same accuracy all the data. **If all the data are estimated with the same accuracy, then, most likely some of the most critical data may be estimated with less accuracy than some of the least critical data**.

Current research by the author has revealed some intriguing results. These findings form the main motivation for further investigating the sensitivity analysis issue in MCDM problems. Consider the previous decision matrix. When one is asked to identify the most critical decision criterion, the intuitive answer is to point to the criterion with the highest weight C_j (for $j=1,2,3,\ldots, N$). However, by doing so one identifies the most **important criterion**, which may or may not be the most **critical criterion**. Recall, that the most critical criterion is the one for which the smallest change in the value of its current weight, will cause a change on the current ranking of the alternatives. This issue was illustrated in the previous numerical example. In [Triantaphyllou and Sanchez, 1994a and 1994b] it was found on simulated test problems that very often in random MCDM problems the most critical criterion is the one with the lowest weight (i.e., the **least important** criterion!).

As Dantzig [1963] stated it: *"Sensitivity analysis is a fundamental concept in the effective use and implementation of quantitative decision models, whose purpose is to assess the stability of an optimal solution under changes in the parameters"*. In general, there is considerable research on this issue regarding several decision-making models, such as linear programming and investment analysis. However, research on sensitivity analysis in deterministic multi-criteria decision making models is limited. Some related work can be found in [Barron and Schmidt, 1988], [Watson and Buede, 1987], [Von Winterfeldt and Edwards, 1986], [Evans, 1984], [Ríos, 1990], and [Masuda, 1990].

The main problem which is defined in this section is ***how to determine the most critical criterion*** in the previous decision making problem. Intuitively, one may think that the **most critical criterion** is the criterion which has the **highest weight** W_j. However, this notion of criticality may be **misleading**. As it was also mentioned in the introduction section, there are two related concepts: one is the concept of the most **significant** criterion, and the other is the concept of the most **critical** criterion. As it was stated earlier, these definitions are distinct. In this project, the ***most critical criterion is the one for which the smallest change in its current weight will alter the existing ranking of the alternatives***. In the previous paragraph the notion of criticality, the term *"smallest change"* will be defined in **absolute** and also in **relative terms** (that is, % of change from its current value).

Changes on the existing ranking of the alternatives can also be viewed from **two** different perspectives. One might be interested in seeing when **any** two alternatives reverse their existing ranking. However, it is also possible one to be interested in **only when the best** alternative changes. The following abbreviations and terms are used in this proposal to indicate four alternative ways of defining the most critical criterion:

1. *The **Best-Absolute-Terms (BAT)** critical criterion*
2. *The **Any-Absolute-Terms (AAT)** critical criterion*
3. *The **Best-Relative-Terms (BRT)** critical criterion*
4. *The **Any-Relative-Terms (ART)** critical criterion.*

Let $\delta_{k,i,j}$ ($1 \leq i < j = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, n$) denote the minimum change in the current weight W_k of criterion C_k such that the ranking of alternatives A_i and A_j will be reversed. Then, the previous types of critical criteria can be described more formally in the following definitions:

DEFINITION 1:
The Best-Absolute-Terms (BAT) critical criterion is the criterion which corresponds to the smallest $\delta_{k,1,j}$ ($j=2,3,\ldots,m$ and $k=1,2,3,\ldots,n$) value, when the $\delta_{k,1,j}$ value is measured in terms of:

a) change of the ranking of the best alternative
and *b) absolute change of the weights.*

DEFINITION 2:
The Any-Absolute-Terms (AAT) critical criterion is the criterion which corresponds to the smallest $\delta_{k,i,j}$ *(*$1 \leq i < j = 1,2,3,\ldots,m$ *and* $k=1,2,3,\ldots,n$*) value, when the* $\delta_{k,i,j}$ *value is measured in terms of:*
a) change of the ranking of any alternative
and *b) absolute change of the weights.*

The definitions for the BRT and ART critical criteria, are analogous to the previous two definitions.

The main goal of a sensitivity analysis of the previous type is to determine how critical each criterion is. In other words, how **sensitive** the current ranking of the alternatives is to changes of the current weights of the decision criteria. In this way the most critical criteria can be identified. This sensitivity analysis will determine what is the **smallest change** in the current weights of the criteria, which can alter the existing ranking of the alternatives.

The intuitive belief is that the criterion with the highest weight is the most critical one [Wayne 1991]. However, this approach may be **seriously misleading**. In research done by the author and one of his Ph.D. students (described in [Triantaphyllou and Sánchez, 1994a]) a sensitivity analysis on the weights of the criteria was performed for the case of the analytic hierarchy process. That investigation considered a number of random test problems and it was found that very often the criterion with the lowest weight might be the most critical one, while the criterion with the highest weight might be the least critical one. In a later study by the same authors [Triantaphyllou and Sánchez, 1994b] the sensitivity of the criteria weights was examined from a more comprehensive point of view for the WSM, WPM, AHP, and the Ideal Mode AHP methods.

6. Determining the Most Critical Criterion

This section presents the proposed methodology for determining the most critical criterion. It applies the previous definitions of the most critical criterion with the WSM and AHP methods.

In [Triantaphyllou and Sanchez, 1994b] the following theorem was demonstrated to hold regarding the impact on the current ranking of alternatives of changes on the weights of the decision criteria when the WSM, AHP, or the Ideal Mode AHP are used.

THEOREM:
When the WSM, AHP or the Ideal Mode AHP methods are used, the critical quantity $\delta_{k,i,j}$ *(*$1 \leq i < j \leq m$ *and* $k = 1, 2, \ldots, n$*), by which the current weight* W_k *of the criterion* C_k *needs to be modified so that the ranking of the alternatives* A_i *and* A_j *will be reversed, is given as follows:*

$$\delta_{k,i,j} < \frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \quad \textit{when } (a_{jk} > a_{ik}) \ \textit{ or:}$$

$$\delta_{k,i,j} > \frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \quad \textit{when } (a_{jk} < a_{ik}) \ .$$

Furthermore, the following constraint should also be satisfied:

$$W_k - 1 \leq \delta_{k,i,j} \leq W_k \ .$$

The above theorem was used to derive the results depicted in table 2, in section 5.

When a first estimate of a decision matrix is given, along with the corresponding variances of the data elements, a sensitivity analysis on the a_{ij} and W_j values should be performed. The next portion of the budget will be allocated according to two factors: *i)* How critical a piece of data is in ranking changes, and *ii)* What is its current variance. That is, the higher the criticality and the higher the variance, the more money should be allocated to estimate that piece of data (and vice-versa). The amount to be allocated for the first estimation of the data needs to be studied as well, possibly empirically. The same is true for the number of steps and the portion of the budget to be allocated to each step.

7. Conclusions and Discussion

The paramount importance of MCDM in solving many real life problems combined with a systematic sensitivity analysis methodology for these problems, make urgent the need for enhancing existing MCDM methods. Moreover, often data in MCDM problems are difficult to be quantified and are also easily changeable. This realization adds to the need for developing effective sensitivity analysis approaches.

Often real life MCDM problems require the acquisition of very large amounts of heterogeneous data. What makes the situation even more challenging is the fact that these data can be very difficult to be quantified. Also, data can be dynamic. If the decision maker(s) spends all their budget in determining these data with the **same accuracy**, then it is possible that **critical** data of the problem are **not known with adequate accuracy**, while **unimportant** data are known with **too much accuracy**.

A decision analysis in the early stage of the data acquisition phase can provide the decision maker with some valuable insights. At an early stage all the data may be gathered with just some acceptable accuracy. Next, a sensitivity analysis is performed. In a second phase, more emphasis is given in **refining** the values of the data which appear to be more critical and with higher accuracy. Next, a new sensitivity analysis is performed and in turn the data are, again, refined according to their criticality and variances. This **stepwise process** can be iterated until the entire budget is spent or the decision maker feels comfortable with the robustness of the decision problem.

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