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Ranking Irregularities when Evaluating Alternatives by Using Some Multi-Criteria Decision Analysis Methods

By Xiaoting Wang[1] and Evangelos Triantaphyllou[2]

[1]Dept. of Industrial Engineering, 3128 CEBA Building
[2]Dept. of Computer Science, 298 Coates Hall
Louisiana State University, Baton Rouge, LA 70803, U.S.A.
Email: xwang8@lsu.edu and trianta@lsu.edu
Webpage: http://bit.csc.lsu.edu/~xiaoting
http://www.csc.lsu.edu/trianta

Abstract:

This chapter first introduces the main issues of multi-criteria decision analysis (MCDA). This involves discussion on some well-known MCDA methods. Next it describes some ranking irregularities when some MCDA methods are used. Ranking irregularities occur when certain manipulations on the structure of a simple MCDA problem are performed. Though a plethora of MCDA methods have been proposed to analyze the data of a decision problem and rank the alternatives, often times different MCDA methods may yield different answers for exactly the same problem. There is no exact way to know which method gives the right answer. This situation leads to the question of how to evaluate the performance of different MCDA methods. To partially answer this question, three test criteria based on some past related studies are presented.

Keywords: Multi-criteria decision analysis, ranking irregularities, rank reversal, ELECTRE methods, the Analytic Hierarchy Process (AHP), utility functions.

1 Introduction to MCDA

People make decisions almost everyday and everywhere. For common individuals, they seldom need to use sophisticated decision-making tools when making their decisions. In many fields of engineering, business, government, and sciences, where decisions often times are either worth millions of dollars or may have a significant impact on the welfare of the society, decision-making problems are usually complex and anything but easy tasks to be completed. In such settings powerful decision analysis and decision-making tools must be built and used to help decision makers make better choices.

There are many decision-making tools in the literature. Some focus on inventory control, investment selection, scheduling etc. Among them, multi-criteria decision analysis (MCDA) is one of the most widely used decision methodologies. MCDA can help to improve the quality of decisions by making decision-making more explicit, rational, and efficient.

A typical problem in MCDA is concerned with the task of ranking a finite set of decision alternatives, each of which is explicitly described in terms of different characteristics (also often called attributes, decision criteria, or objectives) which have to be taken into account simultaneously. Usually, an MCDA method aims at one of the following four goals, or "problematics" [Roy, 1985], [Jacquet-Lagreze and Siskos, 2001]:

Problematic 1: Find the best alternative.

Problematic 2: Group the alternatives into well-defined classes.

Problematic 3: Rank the alternatives in order of total preference.

Problematic 4: Describe how well each alternative meets all the criteria simultaneously.

Many interesting aspects of MCDA theory and practice are discussed in [Hobbs, 1986], [Hobbs, *et al.*, 1992, 2000], [Stewart, 1992], [Triantaphyllou, 2000], [Zanakis, *et al.*, 1995], and [Zanakis, *et al.*, 1998].

Another term that is also used frequently to mean the same type of decision models is multi-criteria decision-making (MCDM). It should be stated here that the term MCDM is also used to mean finding the best alternative in a continuous setting.

Although different MCDA methods follow different procedures, almost all of them share the following common essentials. That is, a finite set of alternatives and a finite set of decision criteria. Each alternative is, somehow, described by how well it meets each one of the decision criteria. If a given criterion refers to a qualitative aspect of the alternatives, then the alternatives may be described in relative or qualitative terms regarding that criterion. In case the criterion is easily quantifiable, then the alternatives may be described in absolute terms regarding that criterion. Meanwhile, the criteria may be associated with weights of importance.

For example, in the hypothetical problem of selecting the best car among three candidate cars, say car A, car B and car C, the decision criteria may refer to price, mileage per gallon, and the physical attractiveness of the shape of a car. That is, we have three criteria. Of these three criteria, the first two are easy to quantify as one may have the exact price value of each car and also the exact fuel consumptions. On the other hand, expressing the alternatives in terms of the last criterion might be trickier as that criterion is a qualitative one. In such cases one may use percentages expressing how much a given car is more desirable than another car.

The above data can also be viewed as the entries of a *decision matrix*. The rows of such a matrix correspond to the alternatives of the problem while the columns to the decision criteria. The a_{ij} element of a decision matrix represents the performance value of the *i-th* alternative in terms of the *j-th* criterion. The typical decision matrix can be represented as in Figure 1 (observe that the criteria weights are depicted in this matrix as the w_j parameters). Data for MCDA problems can be determined by direct observation (if they are easily quantifiable) or by indirect means if they are qualitative [Triantaphyllou, *et al.*, 1994] as we have demonstrated in the previous car selection example.

$$
\begin{array}{lcccc}
 & & \textbf{C r i t e r i a} & & \\
 & C_1 & C_2 & \dots & C_n \\
 & (w_1 & w_2 & \dots & w_n) \\
\textbf{Alternatives} & & & & \\
\cline{2-5}
A_1 & a_{11} & a_{12} & \dots & a_{1n} \\
A_2 & a_{21} & a_{22} & \dots & a_{2n} \\
 & . & . & & . \\
 & . & . & & . \\
 & . & . & & . \\
A_m & a_{m1} & a_{m2} & \dots & a_{mn}
\end{array}
$$

Figure 1. Structure of a Typical Decision Matrix.

From the early developments of the MCDA theories in the 1950s and 1960s, a plethora of MCDA methods have been developed in the literature and new contributions are continuously coming forth in this area. There are also many ways to classify the existing MCDA methods. One of the ways is to classify MCDA methods according to the type of data they use. Thus, we have deterministic, stochastic, or fuzzy MCDA methods [Triantaphyllou, 2000]. Another way of classifying MCDA methods is according to the number of decision makers involved in the decision process. Hence, we have single decision maker MCDA methods and group decision-making MCDA. For some representative articles in this area, see [George, *et al.*, 1992], [Hackman and Kaplan, 1974], and [DeSanctis and Gallupe, 1987]. For a comprehensive presentation of some critical issues in group decision-making, the interested reader may want to consult the papers regularly published in the journal Group Decision Making. In this chapter we concentrate on single decision maker deterministic MCDA methods which attempt to find the best alternative subject to a finite number of decision criteria.

This chapter is organized as follows. The second section presents some well-known MCDA methods. Applications of MCDA methods in different engineering fields are described in the third section. The fourth section discusses various ranking issues that emerge when evaluating alternatives by using different MCDA methods. Finally, some concluding comments are presented in the last section.

2 Some MCDA methods

Among the numerous MCDA methods, there are several prominent families that have enjoyed a wide acceptance in the academic area and many real-world applications. Each of these methods has its own characteristics, background logic and application areas. Next we will give a brief description for some of them.

2.1 The Analytic Hierarchy Process and Some of Its Variants

The Analytic Hierarchy Process (or AHP) method was developed by Professor Thomas Saaty [Saaty, 1980; and 1994]. It is a powerful decision-making process which can help people set priorities and choose the best options by reducing complex decision problems to a system of hierarchies. Since its inception, it has evolved into several different variants and has been widely used to solve a broad range of multi-criteria decision problems. The applications can be found in business, industry, government, and military.

2.1.1 The Analytic Hierarchy Process

The AHP method uses the pairwise comparisons and eigenvector methods to determine the a_{ij} values and also the criteria weights w_j. The details about the pairwise comparisons and eigenvector methods can be found in [Saaty, 1980; and 1994]. In this method, a_{ij} represents the relative value of alternative A_i when it is considered in terms of criterion C_j. In the original AHP method, the a_{ij} values of the decision matrix need to be normalized vertically. That is, the elements of each column in the decision matrix add up to one. In this way, values with various units of measurement can be transformed into dimensionless ones. If all the criteria express some type of benefit, then according to the original AHP method, the best alternative is the one that satisfies the following expression:

$$P^*_{AHP} = \max_i P_i = \max_i \sum_{j=1}^{n} a_{ij} w_j \,, \qquad \text{for } i = 1, 2, 3, ..., m. \tag{2-1}$$

From the above formula, we can see that the original AHP method uses an additive expression to determine the final priorities of the alternatives in terms of all the criteria simultaneously. Next we consider the revised AHP, which is an additive variant of the original AHP method.

2.1.2 The Revised Analytic Hierarchy Process

The revised AHP model was proposed by Belton and Gear in [1983] after they had found a case of ranking abnormality that occurred when the original AHP was used. In their case, the original AHP was used to rank three alternatives in a simple test problem. Then a fourth alternative, identical to one of the three alternatives, was introduced in the original decision problem without changing any other data. The ranking of the original three alternatives was changed after the revised problem was ranked again by the same method. Later this ranking abnormality was defined as a rank reversal. According to Belton and Gear the root for this inconsistency is the fact that the relative values of the alternatives for each criterion sum up to one. So instead of having the relative values of the alternatives sum up to one, they proposed to divide each relative value by the maximum value of the relative values. According to this variant, the a_{ij} values of the decision matrix need to be normalized by dividing the elements of each column in the decision matrix by the largest value in that column. As before, the best alternative is given again by the additive formula (2-1), but now the normalization is different.

$$P^{*}_{Revised-AHP} = \max_{i} P_i = \max_{i} \sum_{j=1}^{n} a_{ij} w_j \,, \quad \text{for } i = 1, 2, 3, ..., m.$$

The revised AHP was sharply criticized by Saaty in [1990]. After many debates and a heated discussion (e.g., [Dyer, 1990a; and 1990b], [Saaty, 1983; 1987; and 1990], and [Harker and Vargas, 1990]), Saaty accepted this variant and now it is also called the *ideal mode* AHP [Saaty, 1994]. However, even earlier, the revised AHP method was found to suffer of some other ranking problems even without the introduction of identical alternatives [Triantaphyllou and Mann, 1989]. In that study and also in [Triantaphyllou, 2000; and 2001], it was found that most of the problematic situations of the AHP methods are caused by the required normalization (either by dividing by the sum of the elements or by the maximum value in a vector) and the use of an additive formula on the data of the decision matrix for deriving the final preference values of the alternatives. However, in the core step of one of the MCDA methods known as the Weighted Product Model (WPM) [Bridgeman, 1922; Miller and Starr, 1969], the use of an additive formula is avoided by using a multiplicative expression. This brought the development of a multiplicative version of the AHP method, known as the multiplicative AHP.

2.1.3 The Multiplicative Analytic Hierarchy Process

The use of multiplicative formulas in deriving the relative priorities in decision-making is not new [Lootsma, 1991]. A critical development appears to be the use of multiplicative formulations when one aggregates the performance values a_{ij} with the criteria weights w_j. In the WPM method, each alternative is compared with others in terms of a number of ratios, one for each criterion. Each ratio is raised to the power of the relative weight of the corresponding criterion. Generally, the following formula is used ([Bridgeman, 1922; Miller and Starr, 1969]) in order to compare two alternatives A_K and A_L:

$$R\left(\frac{A_K}{A_L}\right) = \prod_{j=1}^{n} \left(\frac{a_{Kj}}{a_{Lj}}\right)^{w_j} \tag{2-2}$$

If R (A_K / A_L) $\geq$ 1, then A_K is more desirable than A_L (for the maximization case). Then the best alternative is the one that is better than or at least equal to all other alternatives.

Based on the WPM method, Barzilai and Lootsma, in [1994] and Lootsma in [1999], proposed the multiplicative version of the AHP method. This method was further analyzed in [Triantaphyllou, 2000; and 2001]. According to this method, the relative performance values a_{ij} and criteria weights w_j are not processed according to formula (2-1), but the WPM formula (2-2) is used instead.

Furthermore, one can use a variant of formula (2-2) to compute preference values of the alternatives that in turn, can be used to rank them. The preference values can be computed as follows:

$$P_{i,multi-AHP} = \prod_{j=1}^{n} \left(a_{ij}\right)^{w_j} \tag{2-3}$$

Please note that if $P_i > P_j$, then $P_i / P_j > 1$, or equivalently, $P_i - P_j > 0$. That is, two alternatives A_i and A_j can be compared in terms of their preference values P_i and P_j by forming the ratios or, equivalently, the differences of their preference values.

From formula (2-2), we can see that not only the use of an additive formula was avoided in the multiplicative AHP, but also the negative effects of normalization can also be eliminated by using the multiplicative formula. These properties of the multiplicative AHP are demonstrated theoretically in [Triantaphyllou, 2000]. In that study, it was also proved that most of the ranking irregularities which occurred when the additive variants of the AHP method were used will not occur with the multiplicative AHP method.

2.2 The ELECTRE Methods

Another prominent role in MCDA methods is played by the ELECTRE approach and its derivatives. This approach was first introduced in [Benayoun, *et al.*, 1966]. The main idea of this method is the proper utilization of what is called "outranking relations" to rank a set of alternatives. The ELECTRE approach uses the data within the decision problems along with some additional threshold values to measure the degree to which each alternative outranks all others. Soon after the introduction of the first ELECTRE method, a number of variants have been proposed. Today the two widely used versions are ELECTRE II [Roy and Bertier, 1971, 1973] and ELECTRE III [Roy, 1978] methods. Since the ELECTRE approach is more complicated than the AHP approach, the process of ELECTRE II is described next for a simple introduction of its logic.

The ELECTRE methods are based on the evaluation of two indices, the *concordance index* and the *discordance index*, defined for each pair of alternatives. The concordance index for a pair of alternatives *a* and *b* measures the strength of the hypothesis that alternative *a* is at least as good as alternative *b*. The discordance index measures the strength of evidence against this hypothesis [Belton and Stewart, 2001]. There are no unique measures of concordance and discordance indices.

In ELECTRE II, the concordance index *C(a, b)* for each pair of alternatives *(a, b)* is defined as follows:

$$C(a,b) = \frac{\sum_{i \in Q(a,b)} w_i}{\sum_{i=1}^{m} w_i}.$$

Where *Q (a, b)* is the set of criteria for which alternative *a* is equal or preferred to (i.e., at least as good as) alternative *b* and w_i is the weight of the *i-th* criterion. One can see that the concordance index is the proportion of the criteria weights allocated to those criteria for which *a* is equal or preferred to *b*. The discordance index *D (a, b)* for each pair *(a, b)* is defined as follows:

$$D(a,b) = \frac{\max_j [g_j(b) - g_j(a)]}{\delta}.$$

Where $\delta = \max_j |g_j(b) - g_j(a)|$ (i.e., the maximum difference on any criterion). This formula can only be used when the scores for different criteria are comparable. After computing the concordance and discordance indices for each pair of alternatives, two outranking relations are built between the alternatives by comparing the indices with two pairs of threshold values. They are referred to as the strong and weak outranking relations.

We define (C^*, D^*) as the concordance and discordance thresholds for the strong outranking relation and (C^-, D^-) as the concordance and discordance thresholds for the weak outranking relation where $C^* > C^-$ and $D^* < D^-$. Then the outranking relations will be built based on the following rules:

(1) If $C(a, b) \geq C^*$, $D(a, b) \leq D^*$ and $C(a, b) \geq C(b, a)$, then alternative a is regarded as strongly outranking alternative b.

(2) If $C(a, b) \geq C^-$, $D(a, b) \leq D^-$ and $C(a, b) \geq C(b, a)$, then alternative a is regarded as weakly outranking alternative b.

The value of (C^*, D^*) and (C^-, D^-) are decided by the decision makers for a particular outranking relation. These threshold values may be varied to give more or less severe outranking relations; the higher the value of $C^*$and the lower the value of D^*, the more severe (i.e., stronger) the outranking relation is. That is, the more difficult it is for one alternative to outrank another [Belton and Stewart, 2001]. After establishing the strong and weak outranking relations between the alternatives, the *descending* and *ascending distillation* processes are applied to the outranking relations to get two pre-orders of the alternatives. Next by combining the two pre-orders together, the overall ranking of the alternatives is determined. For a detailed description of the distillation processes, we refer interested readers to [Belton and Stewart, 2001] and [Rogers, *et al.*, 1999].

Compared with the simple process and precise data requirement of the AHP methods, ELECTRE methods apply more complicated algorithms to deal with the complex and imprecise information from the decision problems and use these algorithms to rank the alternatives. ELECTRE algorithms look reliable and in neat format. People believe that the process of this approach could lead to an explicit and logical ranking of the alternatives. However this may not always be the case. This point is further explored in the fourth section.

2.3 Utility or Value Functions

In contrast with the above approaches, there is another different type of analysis which is based on value functions. These methods use a number of trade-off determinations which form what is known as utility or value functions [Kirkwood, 1997]. The utility or value functions attempt to model mathematically a decision maker's preference structure by a utility function (if the problem is stochastic) or a value function (if the problem is deterministic), and these functions are next used to identify a preferred solution [Al-Rashdan, *et al.*, 1999].

The functions attempt to map changes of values of performance of the alternatives in terms of a given criterion into a dimensionless value. Some key assumptions are made in the process for transferring changes in values into these dimensionless quantities [Kirkwood, 1997]. The roots to this type of analysis can be found in [Edwards, 1977], [Edwards and Barron, 1994], [Edwards and Newman, 1986], and [Dyer and Sarin, 1979].

3 Some Applications of MCDA in Engineering

MCDA methods had long been used in many areas of real-life applications, especially in the engineering world. For example, the ELECTRE methods have been widely used in civil and environmental engineering [Zavadskas, et al., 2004; Hobbs and Meier, 2000]. Part of the related projects includes water resources planning [Raj, 1995], waste water or solid waste management [Rogers and Bruen, 1999; Hokkanen and Salminen, 1997], site selection for the disposal of nuclear waste (nuclear waste management), and highway design selection etc. MCDA methods have also been the main tools that are used to solve many kinds of environmental decision-making problems by the U.S. Department of Energy's Environmental Management in the National Research Council of the U.S.A. Hobbs and Meier, in [2000] presented an extensive study about the applications of MCDA methods in energy and environmental decision-making.

MCDA methods also play a significant role in financial engineering. Its applications within this area have covered many important issues, including venture capital investment, business failure risk, assessment of granting credit and investments, and portfolio management. In [2000] Zopounidis and Doumpos delivered a detailed description about the applications of some MCDA methods in financial engineering and how to combine those methods with some other techniques, like expert systems and artificial intelligence technologies, to address the decision problems in financial engineering.

Industrial engineering is another field where MCDA methods are studied intensively and used extensively. One of the most important contributions of industrial engineering is in assisting people to make sound decisions by scientific and appropriate decision-making tools. Triantaphyllou and Evans in [1999] co-edited an issue of the journal Computers and Industrial Engineering which was specialized on some vital MCDA issues in industrial engineering, including facility layout and location problems, maintenance related decision-making, process planning, and production planning and some theoretic issues about MCDA methods in industrial engineering.

Some other engineering applications of MCDA include the use of decision analysis in integrated manufacturing [Putrus, 1990], in flexible manufacturing systems [Wabalickis, 1988], and in material selection [Liao, 1996]. It is impossible to give an exhaustive review of the applications of MCDA methods in engineering which has accumulated a huge literature in the past quarter century. It should have been clear from the above enumeration that scientific and efficient decision-making methods have played and are playing an important and indispensable role in many decision-making activities related to engineering.

4 Ranking Irregularities when Evaluating Alternatives in MCDA

We have seen that a lot of methods have been proposed to analyze and solve multi-criteria decision-making problems in various fields. However, a hot topic in the MCDM area is that often times different MCDA methods may yield different answers to exactly the same problem. Sometimes some types of ranking irregularities may happen to some well-known MCDA methods, for example, the AHP method.

4.1 Ranking Irregularities when the Additive Variants of the AHP Method are Used

The AHP method has been widely used in many real-life decision problems. Thousands of AHP applications have been reported in edited volumes and books (e.g., Golden, *et al.*, 1989, Saaty and Vargas, 2000) and on websites (e.g., www.expertchoice.com). However, the AHP method has also been criticized by many researchers for some of its problems. One key such problem is rank reversals. Belton and Gear in [1983] first described the problem of rank reversals with the AHP. Their rank reversal example (please refer to section 2) demonstrated that the ranking of alternatives may be alerted by the addition (or deletion) of non-optimal alternatives. This phenomenon inspired some doubts about the reliability and validity of the original AHP method.

After the first report, some other types of ranking irregularities with the original AHP method were also found. Dyer and Wendell in [1985] studied rank reversals when the AHP was used and near copies were considered in the decision problem. In [2000] Triantaphyllou reported another type of rank reversals with the additive AHP methods in which the indication of the optimal alternative may change when one of the non-optimal alternatives is replaced by a worse one. Next in [2001] Triantaphyllou reported two new cases of ranking irregularities when the additive AHP methods are used. One is that the ranking of the alternatives may be different when all the alternatives are compared two at a time and also simultaneously. Another case is that the ranking of the alternatives may not follow the transitivity property when the alternatives are compared two at a time.

As we know, the MCDA problems usually involve the ranking of a finite set of alternatives in terms of a finite number of decision criteria. Often times such criteria may be in conflict with each other. That is, an MCDA problem may involve both benefit and cost criteria at the same time. How to deal with conflicting criteria is another factor that may also cause some ranking irregularities. In [Trtantaphyllou and Baig, 2005], it was found that some ranking irregularities occurred with some additive MCDA methods (which include the additive variants of the AHP method) when two different approaches for dealing with conflicting criteria are used. The two approaches are the benefit-to-cost ratio approach and the benefit-minus-cost approach. It was demonstrated that when the two approaches for aggregating conflicting criteria into two groups are used on the same problem, even when using the same additive MCDA method, one may derive very different rankings of the alternatives. Furthermore, an extensive empirical study revealed that this situation might occur rather intensively in random test problems. The only methods that are immune to these ranking irregularities are two multiplicative MCDA methods: the weighted product model (WPM) and the multiplicative AHP.

Many researchers have also put a lot of effort in explaining the reasons behind the rank reversals and study how to avoid them. Belton and Gear in [1983] proposed the revised AHP method in order to preserve the ranking of the alternatives under the presence of identical alternatives. Saaty in [1987] pointed out that rank reversals were due to the inclusion of duplicates of the alternatives. So he suggested that people should avoid the introduction of similar or identical alternatives. However, other cases were later found in which rank reversal occurred without the introduction of identical alternatives [Triantaphyllou, 2000; and 2001]. Dyer in [1990a] indicated that the sum to unity normalization of priorities makes each one dependent on the set of alternatives being compared. He also claimed that the resulted individual priorities are thus arbitrary, as arbitrary sets of alternatives may be considered in the decision problem. Stam and Silva, in [1997] revealed that if the relative preference statements about alternatives were represented by judgment intervals (that is, the pairwise preference judgments are uncertain (stochastic)), rather than single values, then the rankings resulting from the traditional AHP analysis based on the single judgment values may be reversed and therefore are incorrect. Based on this statement, they developed some multivariate statistical techniques to obtain both point estimates and confidence intervals for the occurrence of certain types of rank reversal probabilities with the AHP method. Yue, *et al.*, in [2004] introduced a grouping method based on direct comparisons between all alternatives. Their method divides the alternatives into groups in such a way that a dominant relationship exists between groups but not among alternatives within each group and a rank reversal will not happen between ranking groups. This method can be used in the situation where just a group ranking is desired. The above references are just part of the research studies that people have done on ranking problems when evaluating alternatives by using various MCDA methods. It is evident that many of these ranking problems have not been fully explained. That means the disputes and studies about this hot topic are still going on in the MCDA area and more studies are needed.

4.2 Some Test Criteria for Evaluating MCDA Methods

Most of the past research studies about ranking irregularities concentrated on the AHP method. There are very few studies that explore the reliability and validity of some other MCDA methods. Does that mean decision makers can trust the other MCDA methods without any questioning of the validity of their answers? The answer is “No”. Usually, decision makers undertake some kind of a sensitivity analysis to examine how the decision results will be affected by changes in some of the uncertain data in a decision problem. For example, is the ranking of the alternatives stable or easily changeable under different set of criteria weights? By this process, decision analysts may better understand a decision problem.

However, another intriguing problem with decision-making methods is that often times different methods may yield different answers (rankings) when they are fed with exactly the same decision-making problem (numerical data). Thus, the issue of evaluating the relative performance of different MCDA methods is naturally raised. This, in turn, raises the question how can one evaluate the performance of different MCDA methods? Since for some problems, it may be practically impossible to know which one is the best alternative, some kind of testing procedures have to be determined. The above subjects, along with some other related issues, have been discussed in detail in [Triantaphyllou and Mann, 1989] and [Triantaphyllou, 2000; and 2001]. In those studies, three test criteria were established to test the relative performance of different MCDA methods. These test criteria are as follows:

Test Criterion #1:

An effective MCDA method should not change the indication of the best alternative when a non-optimal alternative is replaced by another worse alternative (given that the relative importance of each decision criterion remains unchanged).

Suppose that an MCDA method has ranked a set of alternatives in some way. Next, suppose that a non-optimal alternative, say A_k, is replaced by another alternative, say $A_k^{/}$, which is less desirable than A_k. Then, the indication of the best alternative should not change when the alternatives are ranked again by the same method. The same should also be true for the relative rankings of the rest of the unchanged alternatives.

Test Criterion #2:

The rankings of alternatives by an effective MCDA method should follow the transitivity property.

Suppose that an MCDA method has ranked a set of alternatives of a decision problem in some way. Next, suppose that this problem is decomposed into a set of smaller problems, each defined on two alternatives at a time and the same number of criteria as in the original problem. Then all the rankings which are derived from the smaller problems should satisfy the transitivity property. That is, if alternative A_1 is better than alternative A_2, and alternative A_2 is better than alternative A_3, then one should also expect that alternative A_1 is better than alternative A_3.

The third test criterion is similar to the previous one but now one tests for the agreement between the smaller problems and the original un-decomposed problem.

Test Criterion #3:

For the same decision problem and when using the same MCDA method, after combining the rankings of the smaller problems that an MCDA problem is decomposed to, the new overall ranking of the alternatives should be identical to the original overall ranking of the un-decomposed problem.

As before, suppose that an MCDA problem is decomposed into a set of smaller problems, each defined on two alternatives and the original decision criteria. Next suppose that the rankings of the smaller problems follow the transitivity property. Then, when the rankings of the smaller problems are all combined together, the overall ranking of the alternatives should be identical to the original ranking before the problem decomposition.

4.3 Ranking Irregularities When the ELECTRE Methods are Used

The performance of some ELECTRE methods was tested in terms of the previous three test criteria in [Wang and Triantaphyllou, 2004; and 2006]. During those experiments, the three test criteria were used to evaluate the performance of TOPSIS [Hwang and Yoon, 1981], ELECTRE II, and the ELECTRE III methods. In those tests, each one of these three methods failed in terms of each one of these three test criteria. This revealed that the same kinds of ranking irregularities which occurred when the additive AHP methods were used also occurred when those ELECTRE methods were used.

For a deeper understanding about those ranking irregularities, a computational experiment was undertaken in [Wang and Triantaphyllou, 2004; and 2006]. The experimental results demonstrated that those ranking irregularities were rather significant in the simulated decision problems. For instance, in terms of test criterion #1, the ranking reversal rate is up to about 20% with the increase of the number of criteria from 3 to 21 for the ELECTRE III method. Sometimes, the best alternatives will become the second best or even lower than that. In terms of test criterion #2, with the increase of the number of alternatives from 3 to 21, the frequency of violating the transitivity property tends to be 100%. Among those decision problems that follow the transitivity property, it was also very likely that the overall ranking of the alternatives from the smaller problems was partially or completely different from the original overall ranking of the un-decomposed problem.

Though the computational results have revealed that those three types of ranking irregularities occurred frequently in simulated decision problems, ten real-life cases selected randomly from the literature were also studied in order to better understand this situation. The results of this study indicated that the rates of these ranking irregularities were also rather high with those real-life cases. For example, six out of ten cases failed test criterion #1. The rankings of 9 out of 10 case studies did not follow the transitivity property. The only case in which the rankings from the smaller problems did not violate the transitivity property failed to pass test criterion #3.

This is the first time in the literature that rank reversals have been reported with the ELECTRE methods. These findings can be viewed as a wake-up call to people that the methods they already used are not as reliable as they may have expected. More reliable decision-making methods are needed to help people make better decisions.

5 Conclusion and Future Research Directions

From the above ranking problems with the AHP and the ELECTRE methods, it can be seen that it is hard to accept an MCDA method as been accurate all the time though they may play a critical role in many real-life problems. The research work in [Wang and Triantaphyllou, 2004; and 2006] complements previous ones and reveals that even more MCDA methods suffer of ranking irregularities. The ELECTRE methods have been widely used today in practice. However, the ranking irregularities should function as a warning in accepting ELECTRE's recommendations without questioning their validity. Previous and current research indicates that the above ranking irregularities tend to occur when the alternatives appear to be very close to each other. If, on the other hand, the alternatives are very distinctive from each other, then it is less likely that these ranking irregularities will take place. However, one needs a more powerful MCDA method when alternatives are closely related to each other. In section 3 it has been shown how widely MCDA methods have been used in various engineering fields. Decisions in those areas are often worth millions or even billions of dollars and have a great influence in the economy and welfare of society. Thus, when evaluating alternatives by different MCDA methods, the ranking problems are worth a great deal of attention.

As it has been mentioned previously in [Triantaphyllou, 2000; and 2001], it is demonstrated that the multiplicative AHP is immune to all of the above ranking irregularities.

This means the multiplicative AHP can pass all the previous three test criteria. Of course, that does not mean it is perfect. It has been found that it may suffer by some other ranking problems [Triantaphyllou and Mann, 1989]. This method uses a multiplicative formula to compute the final priorities of the alternatives. The multiplicative formula can help it to avoid the distortion from any kind of normalization and also some arbitrary effects introduced by the additive formulas. Thus, an intriguing work for the future is to try to see if a new MCDA method can be designed which combines the good qualities from the multiplicative AHP and some other MCDA methods, and is also immune of any ranking problems. Another direction for future research is to discover more test criteria against which existing and future MCDA methods can be evaluated. Clearly, this is a fascinating area of research and it is of paramount significance to both researchers and practitioners in the multi-criteria decision-making field.

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